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On the qualitative behaviors of solutions of some differential equations of higher order with multiple deviating arguments

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Abstract

We study the boundedness of the solutions to a non-autonomous and non-linear differential equation of second order with two constant deviating arguments. We give two examples to illustrate the main results. By this work, we extend some boundedness results obtained for a differential equation with a constant deviating argument in the literature to the boundedness of the solutions of a differential equation with two constant deviating arguments.

1. Introduction

The qualitative behaviors of solutions of differential equations of second order are extensively discussed and are still being investigated by numerous researchers in the literature. Some works performed on the stability, boundedness and integrability of solutions of certain nonlinear differential equations of second order can be summarized as the following: Ahmad and Rama Mohana Rao [1] studied the stability, boundedness and $L^2[a,\infty)$—boundedness of solutions of certain nonlinear differential equations of second order. Gopalsamy [4] gave uniform asymptotic stability criteria for non-autonomous systems using Lyapunov functions. Kolmanovskii and Myshkis [7] were interested in the stability domain of certain functional differential equations of second order. Kolmanovskii and Nosov [8] examined the direct method of Lyapunov and the

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construction of Lyapunov–Krasovskii functionals. Using these functionals they carried out a stability study of nuclear reactors, viral diseases and the design of adaptive controllers with time lags. Krasovskii [9] showed the application of Lyapunov's method to the problem of stability of delay differential equations of second order with time delay. Burton and Hatvani [2] considered the asymptotic stability of second order ordinary and functional differential equations. Csörgő and Hatvani [3] established sufficient conditions which guarantee either stability or instability for the zero solution of a linear differential equation of second order. Hatvani [5] stated sufficient conditions for the stability of the zero solution of non-autonomous second order nonlinear differential equation by using a Lyapunov function related to the total energy. Hatvani [6] presented various stability results for certain linear and non-linear differential equations of second order. Malyseva [11] gave conditions for the uniform boundedness in the limit of the solutions of a Lienard differential equation. Mureşan [13] investigated the boundedness of the solutions of a Lienard type differential system. The theorems proved guarantee that the solutions are uniformly or uniformly ultimately bounded. Nápoles Valdés [14] studied the boundedness and asymptotic stability of a generalized Liénard equation with forcing term. Saker [15] used the theory of time scales in order to provide sufficient conditions for the boundedness of solutions of second-order forced nonlinear dynamic equations. Sun and Meng [16] studied quadratic integrability and boundedness of all solutions of a second order nonlinear differential equation. Tunç [17–26], Tunç and Şevli [27] and Tunç and Tunç [28] discussed stability, uniform stability, boundedness, uniform boundedness and the asymptotic behavior of solutions of certain second order nonlinear differential equations without and with delay. Zhao [29] is concerned with a non-Lienard system of two differential equations. The sufficient conditions together with the sufficient and necessary conditions for the boundedness of solutions of that system are given. The global asymptotic stability of the zero solution of that system and the convergence of solutions to the origin as $t\to\infty$ are studied too. Zhao et al. [31] gave sufficient conditions to quadratic integrability and boundedness for the solutions of second order non-homogeneous delay differential equations. Wang [32] discussed continuability, boundedness and monotonicity of solutions for a class of second order nonlinear differential equations. Xu [33] interested in quadratic integrability and boundedness for solutions to a class of second-order time lag differential equations.

In 1995, Meng [12] considered the ordinary linear differential equation of second order

$$x''(t)+p(t)x'(t)+[q_1(t)+q_2(t)]x(t)=f(t). \tag{1}$$

With the aid of auxiliary functions and integral inequalities, the author obtained some sufficient conditions under which all solutions of Eq. (1) belong to $L^\infty[a,\infty)$ or $L^\infty[a,\infty)\cap L^2[a,\infty)$.

Later, in 2002, Xu [33] studied the second order time lag nonlinear differential equation:

$$(r(t)x'(t))'+[a(t)+b(t)]x(t)=f[t,x(t),x(\varphi(t))],$$

where the author obtained two results on the boundedness and quadratic integrability of the solutions of this equation.

At the same time, in 2002, Sun and Meng [16] took into consideration the following nonlinear differential equation of second order:

$$(r(t)x'(t))'+p(t)x'(t)+[q_1(t)+q_2(t)]x(t)=f(t,x(t)),$$

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