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Construction of periodic and solitary wave solutions for the complex nonlinear evolution equations

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Abstract

In this paper, we present a functional variable method for finding periodic wave and solitary wave solutions of complex nonlinear evolution equations in mathematical physics and engineering sciences. The proposed technique is tested on the generalized Zakharov equation and higher-order nonlinear Schrödinger equations. The method is straightforward and concise, and it can also be applied to other nonlinear evolution equations in applied mathematics.

1. Introduction

Over the last few decades, directly searching for exact solutions of nonlinear partial differential equations has become a more attractive topic in the physical and nonlinear sciences. The investigation of the travelling wave solutions of nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear phenomena appear in a wide variety of scientific applications such as plasma physics, solid state physics, optical fibers, hydrodynamics, biology, and fluid dynamics.

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The study of NLEEs has been going on for the past few decades [1–15]. There has been a tremendous improvement in this area during this time period. Several NLEEs have been formulated depending on the physical situation. Besides, many such equations are generalized to study their general behaviour so that the special cases are truly meaningful both from the physical and mathematical point of view. Several direct and computational methods have been developed in the last few years with the aim of making further progress in this field, deriving more solutions as well as facilitating the calculations involved. The use of computer symbolic systems such as Mathematica and Maple have so far, helped with reducing the tediousness and complications of the calculations involved.

In recent years, various powerful methods of integrability have been established and developed. These techniques are applied left and right to these various NLEEs to solve them and obtain closed form of solutions of physical relevance. There are various solutions that are obtained by incorporating these techniques of integrability. They are soliton solutions, travelling wave, kinks and, peakons and cuspons just to name a few. The study of nonlinear PDEs mostly yield travelling wave solutions.

To achieve our goal, we organize the paper as follows: In Section 2, we describe functional variable method for finding exact solutions of some complex nonlinear evolution equations. In Sections 3 and 4, we illustrate this method in detail by celebrating the generalized Zakharov and higher-order nonlinear Schrödinger equations. Finally, some important conclusions are given.

2. The functional variable method

The features of this method can be presented as follows [16]. For a given nonlinear partial differential equation (PDE), written in several independent variables as

$$P(u, u_t, u_x, u_y, u_z, u_{xy}, u_{yz}, u_{xz}, \ldots) = 0, \tag{2.1}$$

where the subscript denotes partial derivative, P is some function, and $u\{t, x, y, z, \ldots\}$ is called a dependent variable or unknown function to be determined.

We first introduce the new wave variable as $\xi = k(x+ct)$ or $\xi = x - ct$.

The nonlinear partial differential equation can be converted to an ordinary differential equation (ODE) like

$$Q(U, U', U'', U''', \ldots) = 0. \tag{2.2}$$

Let us make a transformation in which the unknown function U is considered as a functional variable in the form

$$U_\xi = F(U) \tag{2.3}$$

and some successively derivatives of U are

$$\begin{aligned} U_{\xi\xi} &= \tfrac{1}{2}(F^2)', \\ U_{\xi\xi\xi} &= \tfrac{1}{2}(F^2)''\sqrt{F^2}, \\ U_{\xi\xi\xi\xi} &= \tfrac{1}{2}[(F^2)'''F^2 + (F^2)''(F^2)'], \end{aligned} \tag{2.4}$$

where "′" stands for d/dU. The ODE (2.2) can be reduced in terms of U,F and its derivatives upon using the expressions of Eq. (2.4) into Eq. (2.2) gives

$$R(U, F, F', F'', F''', F^{(4)}, \ldots) = 0 \tag{2.5}$$

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