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# Investigation of the heat transfer of a non-Newtonian fluid flow in an axisymmetric channel with porous wall using Parameterized Perturbation Method (PPM)

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**Abstract**

In the present article Parameterized Perturbation Method (PPM) is used to obtain the solutions of momentum and heat transfer equations of non-Newtonian fluid flow in an axisymmetric channel with porous wall for turbine cooling applications. The validity of the results of PPM solution were verified by comparison with numerical results obtained using a fourth order Runge–Kutta method. These comparisons reveal that Parameterized Perturbation Method is a powerful approach for solving this problem. The analytical investigation is carried out for different governing parameters namely, Reynolds number, injection Reynolds number, Prandtl number and power law index. The results show that skin friction coefficient increases with increase of Reynolds number, especially at high Reynolds numbers. Also it can be found that Nusselt number has direct relationship with Reynolds number, Prandtl number and power law index.


## 1. Introduction

The problem of non-Newtonian fluid flow has been attracted a lot of attention in recent years various applications in different fields of engineering specially the interest in heat transfer problems of non-Newtonian fluid flow, such as hot rolling, lubrication, cooling problems and

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**Nomenclature**

| | |
|---|---|
| $A$, $B$ | symmetric kinematic matrices |
| $C$ | specific heat |
| $C_n$ | blade-wall temperature coefficients |
| $\frac{\delta v_m}{\delta x_n}$ | velocity gradients |
| $\frac{\delta a_m}{\delta x_n}$ | acceleration gradients |
| $x_k$ | general coordinates |
| $F$ | velocity function |
| $\overline{k}$ | fluid thermal conductivity |
| NM | numerical method |
| $N$ | power law index in temperature distribution |
| $Re$ | injection Reynolds number |
| $K_r$ | rotation parameter |
| PPM | Parameterized Perturbation Method |
| $p$ | fluid pressure |
| $\rho$ | fluid density |
| Pr | Prandtl number |
| $T$ | temperature |
| $q_n(\eta)$ | temperature function |
| $\tau_{ij}$ | stress tensor component |
| $V$ | injection velocity |
| $u_r$, $u_z$ | velocity components in $r$ and $z$ directions, respectively |
| $_F$ | transformation of $f$ |
| $r$, $\theta$, $z$ | cylindrical coordinate symbols |
| $\phi_k$ | viscosity coefficients |
| $\varphi$ | dissipation function |
| $\eta$ | dimension less coordinates in $z$ direction |

drag reduction was the main reason for this considerable attention. Deburge and Han [1] studied a problem concerning heat transfer in channel flow, which can be considered as an application of the previous works reported by Yuan [2], Kurtcebe [3] and White [4]. Increasing the resistance of the blades against the hot stream around the blades for cooling was interested. However the cooling process gives rise to excess energy consumption which leads to overall decrease of turbine efficiency.

In the heart of all different engineering sciences, everything shows itself in the mathematical relation that most of these problems and phenomena are modeled by ordinary or partial differential equations. In most cases, scientific problems are inherently of nonlinearity that does not admit analytical solution, so these equations should be solved using special techniques. Some of them are solved using numerical techniques [5] and some are solved using the analytical method of perturbation [6]. In the numerical method, stability and convergence should be considered so as to avoid divergence or inappropriate results. In the analytical perturbation method, the small parameter is exerted to the equation. Since there are some limitations with the common perturbation method, and also because the basis of the common perturbation method