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Adaptive pseudospectral methods for solving constrained linear and nonlinear time-delay optimal control problems

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Abstract

In this paper, we first develop an adaptive shifted Legendre–Gauss (ShLG) pseudospectral method for solving constrained linear time-delay optimal control problems. The delays in the problems are on the state and/or on the control input. By dividing the domain of the problem into a uniform mesh based on the delay terms, the constrained linear time-delay optimal control problem is reduced to a quadratic programming problem. Next, we extend the application of the adaptive ShLG pseudospectral method to nonlinear problems through quasilinearization. Using this scheme, the constrained nonlinear time-delay optimal control problem is replaced with a sequence of constrained linear-quadratic sub-problems whose solutions converge to the solution of the original nonlinear problem. The method is called the iterative-adaptive ShLG pseudospectral method. One of the most important advantages of the proposed method lies in the case with which nonsmooth optimal controls can be computed when inequality constraints and terminal constraints on the state vector are imposed. Moreover, a comparison is made with optimal solutions obtained analytically and/or other numerical methods in the literature to demonstrate the applicability and accuracy of the proposed methods.
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1. Introduction

Differential control systems with delays in state or control variables describe various processes in the modeling of real-life phenomena in various fields of applications, such as aerospace engineering, robotics, economics, physics, communication networks, chemical processes, transportation, and power systems [1]. Many papers have been devoted to delayed optimal control problems and the

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derivation of necessary optimality conditions. Kharatishvili [2,3] was the first to provide a maximum principle for optimal control problems with a constant state delay and control problems with pure control delays. Halany [4] proves a maximum principle for optimal control problems with multiple equal delays in state and control variables. Similar results were obtained by Soliman and Ray [5]. Göllmann et al. [6] derived a Pontryagin-type minimum (maximum) principle for optimal control problems with constant delays in state and control variables and mixed control-state inequality constraints.

The presence of delay makes analysis and control design much more complicated. Therefore, time-delay systems are very important to many investigators for their control, stability, and optimization [7–12]. The application of Pontryagin's maximum principle to the optimization of control systems with time delays results in a system of coupled two-point boundary value problem involving both delayed and advanced terms whose exact solution, except in highly special cases, is very difficult. Therefore, the main object of all computational aspects of optimal time-delay systems has been to devise a methodology to avoid the solution of the mentioned two-point boundary value problem. Let us divide these methodologies to the solution methods for linear and nonlinear problems. We review some papers concerning different solution techniques. These techniques for linear time-delay optimal control problems include, averaging approximations [13], hybrid of block-pulse functions and Legendre polynomials [14,15], general Legendre wavelets [16], Haar wavelets approach [17], orthonormal basis [18], linear Legendre multiwavelets [19], hybrid of block-pulse functions and Bernoulli polynomials [20] and a composite Chebyshev finite difference method [21]. Moreover, a number of solution techniques have been proposed for nonlinear time-delay optimal control problems which include, Euler discretization method [6], linear semigroup approximation [22], combined parameter and function optimization algorithm [23], iterative dynamic programming [24], semi-infinite programming approach [25], Chebyshev pseudospectral approach [26], the control parametrization enhancing transform [27], control prarameterization [28], an embedding process that transfers the problem to a new optimal measure problem [29], and a neighboring optimal feedback control scheme [30].

During the past few years, pseudospectral methods have been successfully used to solve a wide variety of optimal control problems, owing to its high order of accuracy. Pseudospectral methods are a class of direct collocation where the optimal control problem is transcribed to a mathematical programming problem by parameterizing the state and control using polynomials and collocating the system dynamics using nodes obtained from a Gaussian quadrature, see, e.g., [31–35]. The basis functions are typically Chebyshev, Legendre or even nonclassical orthogonal polynomials [36]. The approaches based on pseudospectral methods can be divided into two categories. In one hand, *global* pseudospectral methods use global polynomials together with Gaussian quadrature collocation points which is known to provide accurate approximations that converge exponentially for problems whose solutions are smooth over the whole domain of interest [34]. However, many optimal control problems have either nonsmooth solutions or nonsmooth problem formulations for which global pseudospectral methods do not provide a satisfactory approximation. On the other hand, *adaptive* pseudospectral methods increase the utility of pseudospectral methods while attempting to maintain as close to exponential convergence as possible. They allow the number of subintervals, subinterval widths, and polynomial degrees to vary throughout the time interval of interest, therefore, they are highly suited for nonsmooth solutions. Some recent progress in this respect have been summarized in [37].

As the solution of a time-delay optimal control problem globally depends on its history due to the delay variable, the exact solutions of these problems are different functions on distinct subintervals. In fact, the optimal control of time-delay systems in natural manner is a bounded and piecewise continuous function. Consequently, in general, the computed response of time-delay

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