Available online at www.sciencedirect.com

ScienceDirect

Journal of the Franklin Institute 351 (2014) 840–856

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Modelling and optimal control of a time-delayed switched system in fed-batch process

Chongyang Liu*, Zhaohua Gong

School of Mathematics and Information Science, Shandong Institute of Business and Technology, Yantai, China

Received 27 May 2012; received in revised form 7 February 2013; accepted 16 September 2013
Available online 4 October 2013

Abstract

The main control goal of the fed-batch process is to maximize the yield of target product as well as to minimize the operation costs simultaneously. Considering the existence of time delay and the switching nature in the fed-batch process, a time-delayed switched system is proposed to formulate the 1, 3-propanediol (1,3-PD) production process. Some important properties of the system are also discussed. Taking the switching instants and the terminal time as the control variables, a free terminal time delayed optimal control problem is then presented. Using a time-scaling transformation and parameterizing the switching instants into new parameters, an equivalently optimal control problem is investigated. A numerical solution method is developed to seek the optimal control strategy by the smoothing approximation method and the gradient of the cost functional together with that of the constraints. Numerical results show that the mass of target product per unit time at the terminal time is increased considerably.

1. Introduction

Recently, fermentation processes have been developed for the manufacture of a vast range of materials from chemically simple feedstocks up to highly complex protein structures [1]. Fermentation processes are fairly difficult to control. This is due to the need for accurate control resulting from the sensitivity of microorganisms and the inability to fully influence internal environment of the cell [2].

*Corresponding author. Tel.: +86 13685358759.
E-mail addresses: liu_chongyang@yahoo.com (C. Liu), zhaohuagong@163.com (Z. Gong).

0016-0032/$32.00
http://dx.doi.org/10.1016/j.jfranklin.2013.09.014

In general, a fermentation process can be operated in batch, fed-batch and continuous mode. Among these modes, the fed-batch operation, in which one or more nutrients are supplied to the reactor during the cultivation and products remain in the containment until the end of the run, is preferable to the others [3]. In order to obtain high productivity, a fed-batch reactor must operate efficiently. Most fed-batch operations are currently constantly fed, exponentially fed, and extended fed-batch. Nevertheless, fed-batch processes are often run by a human operator due to the limitations in operating conditions. As a result, the operation is not always uniform and optimal productivity is seldom obtained [4].

It is obvious that a model-based efficient approach is necessary to ensure maximum productivity with the lowest possible cost in fed-batch processes, without requiring a human operator. Optimal control of bioprocesses is performed based on precise mathematical models. In this work, a mathematical model of a fed-batch process with constant feed rate is derived from the model [5], which has large acceptance in describing dissimilation of glycerol to 1, 3-propanediol (1,3-PD) by *Klebsiella pneumoniae*.

The control strategy of a fermentation process is to maximize the final product yield and to minimize the operation costs. Numerous studies have been reported concerning the control of fed-batch processes in the literature. For the fed-batch processes with pulse feed, optimal impulsive control problem was discussed in [6,7]. In contrast, taking the feed process as a continuous process, optimal multistage control problem was presented in [8]. The existence of the optimal control and numerical algorithms were then investigated [8,9]. More recently, exploiting the hybrid nature of the fed-batch process, optimal switching control problem was presented [10]. Then, for the optimal switching control problem, some important properties and numerical solution approaches were discussed in [10–12]. Although the achieved results are interesting, all those control variables in the optimal control problems are the feed rate of substrate. Moreover, time delays are ignored in the above researches. In fact, time delays exist in the process [13,14] since a cell has to undergo some change or growth process for which it needs some time before it reacts with others.

Unlike the previous researches, we focalize an optimal control problem in fed-batch process with constant feed rate, a simple feed mode has been widely applied for the production of many bioproducts [15–17]. In view of the existence of time delay and the switching nature in the fed-batch process, a time-delayed switched system is proposed to formulate the process. Some important properties of this system are also discussed. In order to obtain high productivity as well as to reduce the operation costs simultaneously, an free terminal time optimal control model involving the proposed system and subject to continuous state constraints is presented. By the way, there has been a mounting interest in optimal control of switched systems [18–23]. Nevertheless, the optimal control of time-delayed switched systems has rarely been considered.

Since the presence of free terminal time as well as time delay makes the optimal control problem much more complicated, existing methods, see, for example [24–26], cannot be used directly to solve the free terminal time delayed optimal control problem in this work. It should, moreover, be noted that the involving time-delayed switched system is highly nonlinear. Therefore it is impossible to obtain analytical solutions of the free terminal time delayed optimal control problem and one has to resort to numerical solution method. By a time-scaling transformation, we equivalently transcribe the free terminal time delayed optimal control problem into the one with fixed terminal time. Furthermore, using the approach in [27], the switching instants in the resultant delayed optimal control problem are parameterized as a new parameter vector. Subsequently, we develop a numerical solution method for the optimal control problem in two aspects. On one hand, the smoothing approximation technique is applied to