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Stabilization of coupled linear plant and reaction-diffusion process

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Abstract

Boundary control to stabilize a system of coupled linear plant and reaction-diffusion process is considered. Backstepping transformations with a kernel function and a vector-valued function are introduced to design control laws. For the situation without heat resource, the kernel function and the vector-valued function of the transformation are obtained, and an explicit control law is established, and simulation results are presented through figures. For the general situation with heat resource, the existence of the kernel and the vector-valued function of the transformations is shown, and an control law is derived. Stability of the closed loops is achieved for both the situations.

1. Introduction

Control designs of engineering control problems are considered in this paper. In engineering applications, there is the situation that a plant is stabilized by heat. Because of corrosion, high temperature, etc., a controller cannot be directly set on or into the plant, alternatively, an thermal conductivity body is employed to transfer heat and control the plant. The controller is set on the thermal conductivity body, and the plant is indirectly controlled through the thermal conductivity body. Another engineering situation is that the plant is controlled by the heat generated by a chemical reaction or biological fermentation. A compensator (controller) is set on the body of heat source to stabilize the plant.

For both situations, control configuration can be illustrated by Fig. 1. The length of the heat body is assumed to be 1, the control input $U(t)$ is set at the “outside” end of the connecting body,

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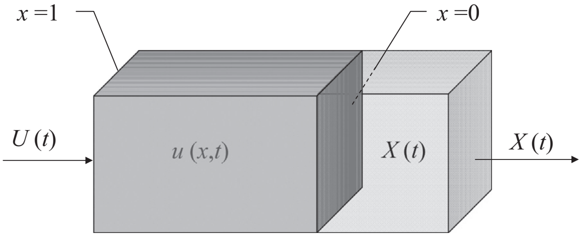

Fig. 1. Control configuration.

namely $x=1$, and the plant is connected to the heat body at $x=0$. Let the plant is modeled by $\dot{X}(t)=AX(t)$, the control configuration illustrated by Fig. 1 is modeled by the following control system:

$$\dot{X}(t)=AX(t)+Bu(0,t) \tag{1}$$

$$u_t(x,t)=u_{xx}(x,t)+\lambda u(x,t), \quad 0<x<1 \tag{2}$$

$$u_x(0,t)=\alpha(u(0,t)-C^TX(t)) \tag{3}$$

$$u(1,t)=U(t) \tag{4}$$

where $X(t)=(X_1(t),X_2(t),\ldots,X_n(t))^T\in\mathbf{R}^n$ is the signal of the plant, and the pair (A,B) is assumed to be stabilizable ($A\in\mathbf{R}^{n\times n}$ and $B\in\mathbf{R}^n$), signal $u(x,t)\in\mathbf{R}$ is the temperature of the thermal body, and the constant λ is non-negative, the Neumann boundary condition (3) comes from the Fourier's Law of Heat Conduction, and $\alpha$is the Fourier constant, which depends on materials of the plant and the heat conductivity body, $C^T\in\mathbf{R}^n$ is a vector such that $C^TX(t)$ is the temperature of the plant, and $U(t)$ denotes the control input.

If the constant $\lambda=0$, the system models the first engineering control problem. If the constant $\lambda>0$, the system models the second engineering control problem. The control objective is to exponentially stabilize entire system signal $(X(t),u(x,t))$ of the closed loop in some sense.

System (2)–(4) is a coupled ordinary differential equation (ODE) and partial differential (PDE). Although there are rich references about the controllability of coupled ODE–PDE systems (see, e.g., Lasiecka [7]), applicable control designs about the coupled ODE–PDE systems have not been established. Recently, Krstic and Smyshlyaev [5,6] developed a backstepping design for boundary control designs of PDEs. Further, Susto and Krstic [4] established a control design for cascaded ODE–PDE systems through the backstepping. Motivated by the design procedures by Krstic and Smyshlyaev [5,6], Tang and Xie [1–3] initiated the control designs of coupled ODE–PDE systems, and established control laws for special class of coupled ODE–PDE systems. In [1,2], the physical configuration is similar to Fig. 1. However, the control system models the first engineering control problem, that is, $\lambda=0$, besides, there is no temperature difference at $x=0$, thus the boundary condition is the Dirichlet one, $u(0,t)=C^TX(t)$. So, the control system (2)–(4) models another class of control problems in engineering. It is valuable to establish its control laws.

The control designs to be established in this paper are as follows. By the backstepping design procedure involved in Susto and Krstic [4] and Tang and Xie [1,2], backstepping transformations which involve kernel functions and vector-valued functions are employed to convert the system into a chosen exponentially stable target systems. Control laws and stabilization are established via the backstepping transformations and its inverses. For the situation that $\lambda=0$, an analytical