Available online at www.sciencedirect.com

ScienceDirect

Journal of the Franklin Institute 351 (2014) 1001–1026

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Observer design for constrained nonlinear systems with application to secure communication

Mohamed F. Hassan*

Electrical Engineering Department, College of Engineering, Kuwait University, 13060, Kuwait

Received 6 November 2012; received in revised form 23 September 2013; accepted 1 October 2013
Available online 10 October 2013

Abstract

In this paper, a constrained regularized least square (RLS) state estimator is developed for deterministic discrete-time nonlinear dynamical systems subject to a set of equality and/or inequality constraints. The stability of the estimation error is rigorously analyzed. The proposed estimator is then used to handle the important problem of secure communication. At the transmitting end, the output of the constrained unified chaotic system is used as a chaotic mask to achieve a satisfactory and typical secure communication scheme. The encrypted data signal is injected into the transmitter and simultaneously transmitted to the receiver through a public channel. At the receiving end, the constrained RLS estimator is used to reconstruct the states of the constrained unified chaotic system. Simulation results are presented to show the impact of the imposed constraints on the waveform and the pattern of the generated chaotic signal as well as the ability of the proposed estimator to synchronize the actual and estimated states of the constrained unified chaotic system. Moreover, the proposed estimator is applied to recover discrete signals such as digital images where computer simulation results are provided to show the effectiveness of the proposed estimation scheme.

1. Introduction

Most mathematical models of real systems, physical processes and man made systems are inherently nonlinear, for example biological systems [1,2], chemical and biochemical processes [3], satellite dynamics [4], etc. Many of these systems are also subjected to a set of equality and/or inequality constraints. These constraints are either due to physical limitations or safety requirements. For example, the gas pressure cannot be negative, the concentrations in a chemical

*Tel.: +965 2498 7634; fax: +965 2481 7451.
E-mail addresses: m.fahim@ku.edu.kw, m.f.hassan47@gmail.com

0016-0032/$32.00
http://dx.doi.org/10.1016/j.jfranklin.2013.10.001

process can neither be negative nor exceed the saturation or the pre-specified upper levels, in penicillin fermentation process the concentration of the biomass, the substrate, the product and the process volume are all restricted by upper and lower bounds [3], in active queue management (AQM) routers supporting transmission control protocol flows (TCP) have windows with limited dimensions [5], etc. On the other hand, constraints may be imposed to a system for safety requirements (e.g. speed limitation, maximum frequency deviation in a power systems to maintain system stability, etc.) or to achieve certain objective (e.g. secure data communication).

In almost all practical applications it is very expensive or even impossible to measure the missing system variables in a direct way. Reconstruction of the unmeasured variables from the knowledge of the systems inputs and outputs remains the possible way to achieve this objective. This problem has been addressed in the literature for both deterministic and stochastic linear and nonlinear systems. However, in this paper we consider the state estimation problem for deterministic nonlinear systems.

Although for linear systems, this subject has been widely discussed and the theory is well developed, this is not the case for nonlinear systems. For continuous nonlinear systems, the available nonlinear observer design techniques can be classified into the linear approach, the Lyapunove based method, the method of extended linearization, the Lie-algebraic approach, the sliding mode approach, the high-gain observers, the state dependent Riccati equation (SDR) approach and others [6–9]. However, these techniques are either inapplicable (e.g. the linear approach) [6], or have severe limitations (e.g. the Lyapunove method) [6], or difficult to use (e.g. Lie-algebraic method) [6], or has restrictive conditions to be satisfied, requires excessive computational time in addition to the fact that the results are not unique (e.g. the SDR approach) [8]. The problem becomes worst for nonlinear dynamical systems characterized by their strong nonlinearities. For discrete nonlinear systems, observer design has been the subject of numerous research papers in which different approaches, such as H_∞ observers, high-gain observer, and others, have been used [10–14]. As a result, it is clear that nonlinear observer design has no systematic solution and the construction of nonlinear observer is extremely dependent upon the form of nonlinearities. In addition to this, there are difficulties of existence and analysis of asymptotic converging observers for nonlinear systems. More importantly, none of the above mentioned techniques is capable to solve the nonlinear observer design problem for constrained nonlinear systems. It is, therefore, necessary to look at a general and a systematic procedure to handle the nonlinear observer design problem for both constrained or unconstrained nonlinear systems, characterized by their strong or weak nonlinearities. Such an approach will not only be useful for nonlinear systems control, but also for other research areas and practical applications, such as the important problem of secure data communication.

It is well known that, in the digital world nowadays, the security of digital signal has become very important since the proliferation of wireless production [15,16]. As a result, secure communication has received increasing attention. The introduction of chaos into communication systems offers several opportunities for improvement. This is due to the random nature of chaotic systems. Although chaotic dynamical systems are deterministic, its random-like behavior can be very useful in distinguishing modulation as noise [17]. Compared to conventional communication systems, chaotic communication systems acquire unique features and potential benefits [17–20]. Among these are efficient uses of the bandwidth of communication channel, utilization of the intrinsic nonlinearities in communication devices, large signal modulation for efficient use of carrier power, secure communication, and the dependency of chaotic signal on initial condition, which may be difficult to guess the structure of the generator and to predict the signal over long time interval. Moreover, chaotic systems, although non-stochastic, share many properties of stochastic processes which is a basic requirement of the spread spectrum communications.