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Consensus of linear multi-agent systems based on full-order observer

Zixi Li*, Hiroshi Ishiguro

Department of systems innovation, Osaka University, Osaka 560-8531, Japan

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Abstract

This paper considers distributed consensus problem of multi-agent systems consisting of general linear dynamics with a time-invariant communication topology. A distributed full-order observer type consensus protocol based on relative output measurements of neighbor agents is proposed. It is found that the consensus problem of linear multi-agent systems with a directed communication topology having a spanning tree can be solved if and only if all subsystems are asymptotically stable. Some necessary and sufficient conditions are obtained for ensuring consensus in multi-agent systems. The design technique is based on algebraic graph theory, Riccati inequality and linear control theory. Finally, simulation example is given to illustrate the effectiveness of the theoretical results.

1. Introduction

In recent years, decentralized coordination of multi-agent systems has received compelling attention [10-19] from various scientific communities due to its broad applications in many areas such as spacecraft formation flying, design of distributed sensors networks, cooperative control of robotic teams, and so on. The multi-agent systems are considered as agents forming a network and information is exchanged between these agents through the network. Although each individual agent has limited sensing and computing ability, the networked system can work together in a coordinated fashion to perform complex tasks. For multi-agent systems, a critical problem is to design a distributed control protocol based on local information that makes all

*Corresponding author. Tel.: +81 8042337910.
E-mail address: li.zixi@irl.sys.es.osaka-u.ac.jp (Z. Li).

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agents to reach an agreement on certain global criteria of common interest by negotiating with their neighbors, which is called as the consensus problem.

Recently, much progress has been achieved in the study of constructing conditions for reaching consensus among multi-agent systems. Jadbabie et al. [1] successfully explained the consensus on the heading angles observed by Vicsek [2]. Saber and Murry [3] studied consensus problems for first-order continuous-time systems with time delay. Ren and Beard [4] obtained that the interaction topology has a spanning tree which is critical for a system to achieve consensus. Yu et al. [5] studied some necessary and sufficient conditions for second-order consensus. Wen et al. [21] studied a distributed consensus problem of linear multi-agent systems with discontinuous observations over a time-invariant undirected communication topology and [22] proposed a consensus algorithm to solve the formation control problem of linear multi-agent systems with intermittent communication constraints as well as the consensus tracking problem with switching directed topologies. In order to better tackle the control problems, tools from graph theory [6], stochastic matrix analysis, system theory, Lyapunov theory [7], linear matrix inequality [8], passivity theory [9] have been applied and proved useful.

In this paper, we consider the consensus problem of multi-agent systems under a fixed communication topology in a more general case, where the dynamics of each agent is any order linear control system, rather than first integrators or double integrators. A distributed observer type consensus protocol based on relative output measurements of neighboring agents is proposed. A multi-step algorithm is presented to construct a full-order protocol, by which the consensus of multi-agent systems with a communication topology containing a spanning tree can be achieved. It is found that consensus can be achieved if and only if all subsystems are asymptotically stable and some necessary and sufficient conditions are obtained for ensuring consensus in multi-agent systems.

The rest of this paper is organized as follows. Section 2 contains some basic notations and some useful results of algebraic graph theory. The distributed observe type consensus protocol is discussed in Section 3. In Section 4 numerical example is simulated to verify the theoretical analysis. Conclusions are finally stated.

2. Preliminaries

In this section, some basic notations and some useful results about algebraic graph theory are briefly summarized which are related to our later analysis.

Let $\mathbf{g}=(V,\varepsilon,A)$ be a weighted directed graph of order N, with a node set $V=[v_1,v_2,\ldots,v_n]$, a directed edge set $\varepsilon\subseteq v\times v$, and a weighted adjacency matrix $A=[a_{ij}]\in R^{N\times N}$. An edge (v_i,v_j) in a weighted directed graph denotes that agent j can obtain information from agent i, but not necessarily vice versa. The weighted adjacency matrix A of a weighted directed graph is defined such that a_{ij} is a positive weight if $(v_j,v_i)\in\varepsilon$, while $a_{ij}=0$ if $(v_i,v_j)\notin\varepsilon$. In this paper, only positively weighted directed graphs are considered, i.e., $a_{ij}>0$ if and only if there is a directed edge (v_j,v_i) in graph $\mathbf{g}$.

A directed path is a sequence of edges in a directed graph of the form $(v_{i_1},v_{i_2}),(v_{i_2},v_{i_3}),\ldots$, where $v_{i_j}\in V$. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all of the other node. A root r is a node having the property that for each node v different from r, there is a directed path from r to v.