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Stability analysis of singularly perturbed control systems with actuator saturation ☆

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Abstract

Stability of singularly perturbed systems (SPSs) with actuator saturation is investigated. A set invariance condition with guaranteed stability bound is proposed. Based on this condition, two optimization problems are formulated separately. One is used to get the best estimate of the stability bound with guaranteed basin of attraction, while the other is used to get the best estimate of the basin of attraction with guaranteed stability bound. The proposed algorithms to solve the optimization problems are independent of the singular perturbation parameter and thus well conditioned. Two examples are included to show the advantages and effectiveness of the proposed results.

1. Introduction

Actuator saturation is ubiquitous in practical engineering systems and can deteriorate the performance of control systems. Thus continual efforts have been devoted to address actuator saturation and fruitful results have been reported [1–4]. Unfortunately, a direct application of these results to singularly perturbed systems (SPSs) may lead to ill-conditioned numerical issues

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because of the two-time-scale nature of the systems [5]. This paper will study stability of the following SPS with actuator saturation

$$\begin{bmatrix} \dot{x} \\ \varepsilon\dot{z} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \mathrm{sat}(u), \tag{1}$$

where $\varepsilon > 0$ denotes the singular perturbation parameter, $x \in R^{n_1}$ and $z \in R^{n_2}$ are the state variables, $u \in R^m$ is the control input. $A_{11}, A_{12}, A_{21}, A_{22}, B_1$ and B_2 are constant matrices of appropriate dimensions, and $\mathrm{sat}(\cdot)$ is a componentwise saturation map $R^m \mapsto R^m$ defined by

$$\mathrm{sat}(u_j) = \mathrm{sign}(u_j)\min\{1, |u_j|\}, \quad j = 1, 2, \ldots, m. \tag{2}$$

Stability problem of system (1) involves two indexes. One is the stability bound ε_0, which is the upper bound for the singular perturbation parameter ε such that stability of the system preserves for all $\varepsilon \in (0, \varepsilon_0]$. Stability bound characterizes the robustness of system stability with respect to the perturbation parameter ε. Many methods have been proposed to calculate stability bound of linear SPSs [6–9]. To enlarge the stability bound, numerous controller design methods for SPSs without actuator saturation have been reported [10–14]. On the other hand, it is known that the global stability is difficult to achieve when system (1) is open-loop unstable [15,16]. Thus the basin of attraction of the closed-loop system is also an important stability index. For control systems without two-time-scale behavior, many approaches to estimate or optimize the basin of attraction of the closed-loop system have been proposed by using Lyapunov stability theory and LaSalle's invariance principle [17–19].

Recently, analysis and design of system (1) have attracted much attention [20–24]. In [20], a composite stabilizing controller was designed by combining the reduced-order controllers for the slow and fast subsystems and a convex optimization problem was formulated to estimate the basin of attraction of SPSs. A full-order controller design method which is independent of system decomposition was proposed in [21]. In [22,23], the so-called reduced-order adjoint systems were introduced under the assumption that the state feedback controller gain was given, by which a reduced-order method was proposed to estimate the basin of attraction of SPSs. The results in [20–23] pay more attention to estimating the basin of attraction than computing the stability bound. They can guarantee the existence of the stability bound, but cannot determine the value of the stability bound. Thus in practice, the effectiveness of the controllers and the estimate of the basin of attraction have to be validated by trial and error. To deal with this problem, an ε-dependent controller was designed, which can achieve a desired stability bound and satisfactory estimate of the basin of attraction of SPSs [24]. However, since the controller and the estimate of the basin of attraction depends on the singular perturbation parameter ε, the results are not valid when ε is not available or undergoes an unknown shift.

The aim of this paper is to study the stability of system (1) under a given linear state feedback controller. A set invariance condition is established by using a ε-dependent Lyapunov function. The condition considers both the stability bound and basin of attraction simultaneously. It is shown that a larger stability bound will lead to a smaller basin of attraction. To cope with such a competition, two optimization problems are proposed to optimize one of the stability indexes by fixing the other. Compared with the existing results [20–24], the proposed methods have the following features: (1) the stability bound and basin of attraction are quantitatively addressed simultaneously; (2) the estimate of the basin of attraction is independent of the singular perturbation parameter ε; (3) the best estimate of one of the stability indexes which guaranteed the other is obtained.