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On the observer design problem for continuous-time switched linear systems with unknown switchings

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Abstract

The observer design problem for Switched Linear Systems (*SLS*) subject to an unknown switching signal is addressed in this work. Based on known observability results for *SLS*, an appropriate *SLS* observer is proposed and its convergence is analysed showing that the corresponding estimates converge in finite-time to the *SLS* state. More precisely, the observers of the continuous state evolution and the observers of the switching signal are investigated and their convergence studied separately. The main tool to analyse the observability is the well-known geometric concept of (A, B)-invariant subspaces. The developed *SLS* observers are then applied to construct synchronized chaotic generators based on the *SLS* with chaotic behavior. Finally, an example of a non-trivial chaotic *SLS* and its detailed analysis are presented to illustrate the achieved results.

1. Introduction

The goal of this paper is to address an observer design for Switched Linear Systems (*SLS*) subject to an unknown switching signal. A *SLS* may be viewed as a subclass of the class of

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Hybrid Dynamical Systems (*HDS*) formed by a collection of Linear Systems (*LS*) together with a time dependent exogenous switching signal. The system state evolution is then uniquely determined by a given initial condition and by the mentioned switching signal determining at each time instant the unique active linear system driving the state.

The study of the fundamental properties of *SLS* has received a great deal of attention during the last decade. In particular, *SLS* observability has been thoroughly analysed depending on whether the switching signal is known or unknown.

On one hand, if the switching signal is known, the focus has been on determining the continuous state after a finite number of switchings [1–4]. It has been shown that the observability of each *LS* is not a necessary condition [1,4]. On the other hand, if the switching signal is unknown (for instance when switchings may occur in an unpredictable way), the observability of the continuous phase and the observability of the switching signal [5–8] were shown to be mutually independent properties [7]. Moreover, if the switching signal is unknown, the observability of each *LS* is not sufficient for the overall *SLS* observability [7]. As a matter of fact, the unknown switching signal computation requires the so-called distinguishability property [5] enabling to detect the current evolving *LS* based on the input–output information only. Last but not least, for the unknown switching signal the controlled inputs also play a central role leading to two different distinguishability notions [9]: (i) distinguishability for every nonzero state trajectory [10] and (ii) distinguishability for "almost every" input [6–8]. The latter case, as expected, requires less restrictive conditions to be fulfilled, in particular, the observability of each *LS* is not required.

To be more specific, the unknown switching signal is usually estimated using the so-called "location observer" based on the *SLS* continuous-time input–output information. In [11], a multi-observer structure based on Luenberger observers together with a residual generator (similar to those used in fault detection) is used as a location observer. The reported results, however, require a careful selection of the threshold for the residue generation. Moreover, as noticed in [11] it can occur that the residue remains true after switching to another subsystem, thus missing the detection of switching. In [12], the location observer algorithm that requires the numerical computation of derivatives of inputs and outputs has been proposed as the location observer. Nevertheless, the analysis presented there is restricted to monovariable *SLS*. In [13], the location observer uses a super twisting based step-by-step observer for switched nonlinear systems that can be transformed into the normal form.

However the focus is on autonomous systems and the super twisting based step-by-step observer requires the knowledge of the bound of the state velocity. In [14] the location observer is formed by a set of Luenberger observers with an associated super twisting based differentiator [15] used to obtain the exact error signal which updates the estimate. Using the results on distinguishability for every nonzero state trajectory, the authors of [14] showed the convergence of the observer. Unfortunately, this distinguishability notion requires the observability of each *LS*.

Furthermore, multi-observer structures have been proposed in the framework of supervisory control of a class of *SLS* composed of a *LS* with an unknown parameter [16–18] and in adaptive control [19,20] where the aim is to decide, based on the size of the output estimation error, which candidate controller (from a bank of controllers) should be used in the feedback loop with the process. Unfortunately, as noticed in [21], this scheme cannot be used to recover the switching signal because the smallness of the output estimation error is not sufficient to infer the evolving *LS* [21].