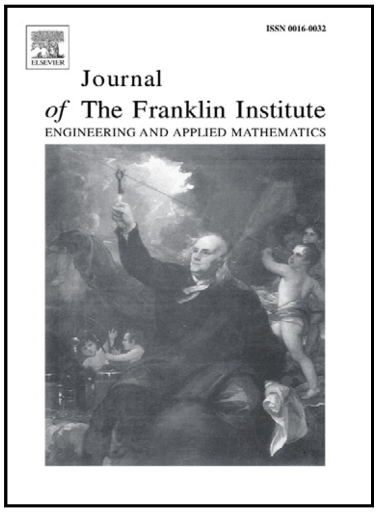

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The minimal rank of matrix expressions with respect to Hermitian matrix–revised

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Abstract

This paper presents representations for the minimal rank of $A - BXC$ with respect to Hermitian matrix X and the minimal rank of $A - BX - YD$ with respect to Hermitian matrix X and Y. These results prove, in a distinctive way, the solvability conditions of the matrix equations $BXC = A$ and $BX + YD = A$, where X and Y are arbitrary matrices with $X = X^*$.

Keywords: minimal rank, Hermitian matrix, Singular value decomposition (SVD), Sylvester's law of inertia, rank formulas

Mathematics Subject Classifications (2010): 15A03; 15A09; 15A24

1 Introduction

We adopt the following notation in this paper. Let $\mathbb{C}^{m\times n}$ denote the set of $m \times n$ matrices with complex entries. The symbols $\mathbb{H}(n)(\mathbb{H}_{\geq}(n))$ and $\mathbb{U}(m)$ stand for the set of all $n \times n$ Hermitian(Hermitian nonnegative-definite) matrices and that of all $m \times m$ unitary matrices, respectively. The *conjugate transpose* of a matrix A is denoted by A^*. For $A \in \mathbb{H}(n)$, $n_+(A)$ and $n_-(A)$ denote the numbers of the positive and negative eigenvalues of $A \in \mathbb{H}(n)$ counted with multiplicities, respectively. The *Moore-Penrose inverse* of $A \in \mathbb{C}^{m\times n}$ is defined as the unique $X \in \mathbb{C}^{n\times m}$ satisfying

$$(1)\ AXA = A, (2)\ XAX = X, (3)\ (AX)^* = AX, (4)\ (XA)^* = XA,$$

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