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Design of parameter-scheduled state-feedback controllers using shifting specifications

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Abstract

In this paper, the problem of designing a parameter-scheduled state-feedback controller is investigated. The paper presents an extension of the classical regional pole placement, $\mathcal{H}_2$ control and $\mathcal{H}_\infty$ control problems, so as to satisfy new specifications, that will be referred to as *shifting pole placement control*, *shifting* $\mathcal{H}_2$ *control* and *shifting* $\mathcal{H}_\infty$ *control*, respectively. By introducing some parameters, or using the existing ones, the controller can be designed in such a way that different values of these parameters imply different regions where the closed-loop poles are situated, or different performances in the $\mathcal{H}_2$ or $\mathcal{H}_\infty$ sense. The proposed approach is derived within the so-called Lyapunov Shaping Paradigm, where a single quadratic Lyapunov function is used for ensuring stability and desired performances in spite of arbitrary parameter time variation. The problem is analyzed in the continuous-time LPV case, even though the developed theory could be applied to LTI systems in cases when it is desired to vary the control system performances online. Results obtained in simulation demonstrate the effectiveness and the relevant features of the proposed approach.

1. Introduction

In the last decades, linear matrix inequalities (LMIs) have emerged as a powerful formulation and design technique for a variety of linear control problems [1]. Indeed, a wide range of

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problems arising in systems and control theory can be reduced to convex optimization problems that involve LMIs, that are appealing from a practical standpoint, since they can be solved by efficient algorithms. Among the problems that lead to LMIs, there are regional pole placement, $\mathcal{H}_\infty$ performance and $\mathcal{H}_2$ performance [2].

The design of control laws that place the closed-loop poles of the system to be controlled at some desired location of the complex plane is one of the most relevant problems in control theory. Historically, the problem of *exact* pole placement, where closed-loop poles are required to lie at some desired location, was studied at the beginning [3]. However, in cases where exact pole placement is not required, *regional* pole placement, where closed-loop poles have to be placed within a prescribed region of the plane, is sufficient. Remarkable results were obtained in [2], where the problem of designing state- or output-feedback H_∞ controllers that satisfy additional constraints on the closed-loop pole location was addressed. Sufficient conditions for feasibility, expressed in terms of LMIs easily tractable from a numerical point of view, were derived for a general class of convex regions of the complex plane. Ref. [2] started from an extension of the Lyapunov characterization of stability made in [4], so as to obtain LMI-based conditions for pole clustering, obtained with the introduction of the so-called *LMI Regions*.

In [5], a parameter-dependent Lyapunov approach has been used to deal with constant or time-varying uncertainty, obtaining less conservative results in the case of slow parametric variations. Further research has aimed to achieve many objectives, among which to enforce robustness [6] or to solve the pole placement problem for filter design [7]. The big amount of articles appeared in the last decade, e.g. [8–13], demonstrates that the regional pole placement problem is still a hot topic of investigation.

On the other hand, a lot of research performed in the last decades has involved two performance measures: the $\mathcal{H}_2$ and the $\mathcal{H}_\infty$ norms, both defined in the frequency-domain for stable transfer matrices. The $\mathcal{H}_2$ optimal control theory was heavily studied in the 1960s as the Linear Quadratic Gaussian (LQG) optimal control theory. On the other hand, the $\mathcal{H}_\infty$ optimal control theory, introduced in an input–output setting [14], reached a mature state in the late 1980s, when it was completed with state space formulations [15] and comprehensive comparisons with the $\mathcal{H}_2$ control problem [16].

Necessary and sufficient conditions for the existence of an $\mathcal{H}_\infty$ controller of any order were given in terms of LMIs in [17,18]. These LMIs correspond to the inequality counterpart of the $\mathcal{H}_\infty$ Riccati equations. Additional flexibility in the $\mathcal{H}_\infty$ control design has been introduced through the use of parameter-dependent quadratic Lyapunov functions in [19]. A systematic design technique that combines the good aspects of both $\mathcal{H}_2$ and $\mathcal{H}_\infty$ methods has been developed later in [20]. Similar to the regional pole placement case, the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems have been the subject of recent research, e.g. [21,22].

Polytopic systems constitute an important field for regional pole placement and $\mathcal{H}_2/\mathcal{H}_\infty$ application. The polytopic representation is one of the most used approaches for describing uncertainty or variability of the parameters in linear parameter-varying (LPV) systems. Looking at [2,5,19,7,21,8,12,13], it can be seen that a lot of effort has been put in developing techniques for regional pole placement and $\mathcal{H}_2/\mathcal{H}_\infty$ control of polytopic systems, taking advantage of some useful properties of LMIs that allow us to assure that the satisfaction of the LMIs at some points, i.e. the vertices of the polytope, implies their satisfaction in all points inside the polytope.

In this work, the problem of designing a parameter-scheduled state-feedback controller is investigated. The main novelty and contribution of this paper is to take advantage of the properties of polytopes and LMIs to solve new problems, that can be seen as extensions of the classical regional pole placement control, $\mathcal{H}_2$ control and $\mathcal{H}_\infty$ control problems, that will be