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Necessary and sufficient conditions for stability of switched nonlinear systems

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Abstract

This paper investigates stability/asymptotic stability of switched nonlinear systems with potentially unstable subsystems. At first, switched nonlinear Hamiltonian-type systems are considered and several necessary and sufficient conditions of stability/asymptotic stability are developed via the maximum energy function based method. Based on the new stability results obtained and the method of Hamiltonian realization, several necessary and sufficient conditions of stability/asymptotic stability are then presented for ordinary switched nonlinear systems with potentially unstable subsystems. A numerical example and its simulations illustrate the effectiveness of the obtained stability results.

1. Introduction

Switched systems, an important class of hybrid systems [1,2], have attracted a lot of attention in control systems community in recent years. Switched systems exist in many practical control systems which need switching among different model structures in their operation processes, for example, power systems in the process of emergency control. Numerous results have been obtained recently for switched systems, see, e.g., [3–24] and references therein. As a fundamental property and one of the most important research topics, stability of switched

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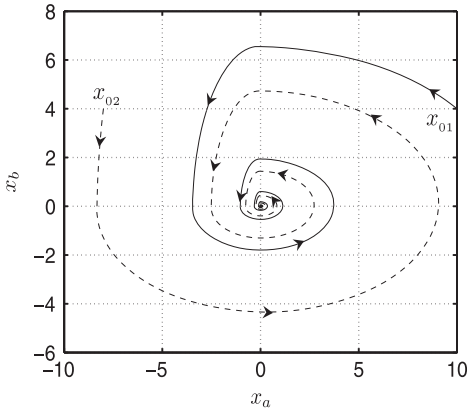

Fig. 1. States of system (103) under λ_1.

systems has been investigated widely and lots of results have been obtained, see, e.g., [3–5,7,11,14,17,16,23,24]. It is well established in [10,14] that if there exists a Common Lyapunov Function (CLF) for all the subsystems of a switched system, then the system is stable under arbitrary switching rules. In [16], some existing necessary and sufficient conditions were presented for the existence of a common quadratic Lyapunov function for a set of stable second-order linear time-invariant systems. A useful CLF algorithm was proposed in [4] for investigating the stabilization of second-order switched linear systems. It is noted that many switched systems do not admit any CLF. To address the problem of the CLF based approaches, many significant efforts have been devoted to investigation of less conservative approaches to stability analysis of switched systems. These approaches include methods based on the so-called Multiple Lyapunov Functions (MLF), Switching Lyapunov Functions (SLF), Multiple Storage Functions (MSF) and the Generalized Multiple Lyapunov-like Functions (GMLF). The well-known MLF based method was first proposed by Peleties and DeCarlo [15] and further extended by Branicky [3]. The MLF method has been proved to be a more powerful and effective tool for both analyzing stability and designing controllers or switching rules for switched systems. The switching Lyapunov function based method [5] is a special case of the MLF based approach. The MSF based method [23] and the GMLF based method [24] aim at further reducing the conservatism of the MLF based approaches. Several stability results based on the MSF and the GMLF methods have been obtained in [23,24], where all subsystems are required to satisfy the *non-increasing condition* (*NC*), that is, the corresponding Lyapunov functions, Lyapunov-like functions and or storage functions are all not increasing during their activating time intervals.

It is well worth pointing out that most stability results [1–3,5,7,8,10,11,14–16,18,23,24] obtained for switched systems are based on the basic assumption that all subsystems are stable or satisfy NC. However, this may not be the case for many real practical switched systems. On the other hand, it is well known that stability of all subsystems does not imply stability of switched systems under arbitrary switching, and that switched systems with all subsystems being unstable might be stable under certain desirable switching rule. So it is desirable to develop some both sufficient and necessary conditions for stability of switched systems if possible. However, this is a challenge, especially for switched systems with all subsystems that are unstable and do not satisfy NC based on existing stability analysis tools, e.g., the CLF, MLF, MSF and GMLF methods. Although there are some results on sufficient conditions for stability of switched

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