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Global state feedback stabilization of high-order nonlinear systems with multiple time-varying delays ☆

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Abstract

This paper studies the global stabilization problem for a class of high-order nonlinear systems with low-order and high-order nonlinearities, and multiple time-varying delays. Systems become more general due to both low-order and high-order in nonlinearities taking values in certain intervals. By introducing a novel Lyapunov–Krasovskii functional, a state feedback controller based on the Lyapunov–Krasviskii theorem together with the adding a power integrator and sign function methods is designed to guarantee the globally uniformly asymptotic stability of the closed-loop system.

1. Introduction

In this paper, we consider a class of high-order nonlinear systems with multiple time-varying delays described by

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}^{p_i}(t) + f_i(t, x(t), x_1(t-\tau_1(t)), \ldots, x_n(t-\tau_n(t))), \quad i = 1, \ldots, n-1, \\ \dot{x}_n(t) &= u^{p_n}(t) + f_n(t, x(t), x_1(t-\tau_1(t)), \ldots, x_n(t-\tau_n(t))), \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^\top \in R^n$ and $u(t) \in R$ are the system state and control input, respectively. For $i = 1, \ldots, n$, $\tau_i(t) : R^+ \to R^+$ is the time-varying delay with $0 < \tau_i(t) \le \varepsilon_i$, where

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ε_i is a positive constant, $p_i \in R_{odd}^{\geq 1} \triangleq \{p/q \in R^+ : p$ and q are odd integers, $p \geq q\}$, $f_i : R^+ \times R^n \times R^n \to R$ is an unknown continuous function with $f_i(t,0,0)=0$. The initial condition is $x(\theta)=\xi_0(\theta)$, $\forall \theta \in [-\tau, 0]$ with $\tau \geq \max\{\varepsilon_1, \ldots, \varepsilon_n\}$ and $\xi_0(\cdot)$ being a specified continuous function. System (1) is called as high-order system if there exists at least one $p_i > 1$ $(1 \leq i \leq n)$. The satisfactory solution to the stabilization problem of system (1) plays a guiding role in some physical systems, such as the underactuated unstable mechanical system in [1], chemical reactor systems with delayed recycle streams in [2,3].

In particular, for high-order nonlinear system (1) without time-delay, i.e.,

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}^{p_i}(t) + f_i(t,x(t)), \quad i=1,\ldots,n-1, \\ \dot{x}_n(t) &= u^{p_n}(t) + f_n(t,x(t)), \end{aligned} \tag{2}$$

many results on feedback stabilization of nonlinear system (2) have been achieved in the past decades, see [4–8] and the references therein. Most of the existing results require that the nonlinearity f_i satisfies a certain restrictive condition, i.e., the states in the bounding function are of an order equal to $1/(p_j \cdots p_{i-1})$, or greater than $1/(p_j \cdots p_{i-1})$, or less than $1/(p_j \cdots p_{i-1})$, e.g., see [9–18] and the references therein.

Recently, the restrictive condition is relaxed by [19–22], in which all the states in the bounding condition are allowed to be of both an order greater than $1/(p_j \cdots p_{i-1})$ and an order equal to $1/(p_j \cdots p_{i-1})$. These assumptions can be summarized in the following form:

$$|f_i(t,x(t))| \leq M \sum_{j=1}^{i} (|x_j(t)|^{\nu_{lj}} + |x_j(t)|^{\nu_{uj}}), \quad i=1,\ldots,n, \tag{3}$$

where low-order $\nu_{lj} = 1/(p_j \cdots p_{i-1})$ and high-order $\nu_{uj} = (g_i + \omega_2)/g_j$ are some ratios of odd integers in $[1/(p_j \cdots p_{i-1}), +\infty)$ with $g_1 = 1$, $g_{i+1} = (g_i + \omega_2)/p_i$ and $\omega_2 \geq 0$.

For the special case of $p_i = 1$, [23,24] weaken growth condition (3) by allowing both low-order $0 < \nu_{lj} \leq 1$ and high-order $1 \leq \nu_{uj} < +\infty$, i.e.,

$$\begin{aligned} |f_i(t,x(t))| &\leq M \sum_{j=1}^{i} \left(|x_j(t)|^{(1+i\omega_1)/(1+(j-1)\omega_1)} + |x_j(t)|^{(1+i\omega_2)/(1+(j-1)\omega_2)} \right), \\ & -\frac{1}{n} < \omega_1 \leq 0, \quad \omega_2 \geq 0. \end{aligned}$$

However, for the general case of $p_i \geq 1$, no further result on the stabilization of nonlinear system (2) is achieved to relax the condition (3) until now.

For high-order nonlinear system (1) with time-delay, several results [25–27] have been achieved on feedback stabilization. But their growth conditions on f_i only have high-order terms.

From the above discussion, a very interesting problem is proposed immediately:

For high-order time-delay nonlinear system (1), *under the condition*

$$\begin{aligned} &|f_i(t,x(t),x_1(t-\tau_1(t)),\ldots,x_n(t-\tau_n(t)))| \\ &\leq M \sum_{j=1}^{i} (|x_j(t)|^{\nu_{lj}} + |x_j(t)|^{\nu_{uj}} + |x_j(t-\tau_j(t))|^{\nu_{lj}} + |x_j(t-\tau_j(t))|^{\nu_{uj}}), \quad i=1,\ldots,n, \end{aligned} \tag{4}$$

is it possible to relax condition (4) *by allowing low-order ν_{lj} and high-order ν_{uj} to take any value in* $(0, 1/(p_j \cdots p_{i-1})]$ *and* $[1/(p_j \cdots p_{i-1}), +\infty)$, *respectively*? *Under the weaker condition, can a stabilized state feedback controller be designed for system* (1)?

In this paper, by introducing a novel Lyapunov–Krasovskii (L–K) functional together with a combined sign function design and the adding a power integrator, we solve the above problem. The main difficulties in the design and analysis are: