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VADER: A flexible, robust, open-source code for simulating viscous thin accretion disks

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ABSTRACT

The evolution of thin axisymmetric viscous accretion disks is a classic problem in astrophysics. While models based on this simplified geometry provide only approximations to the true processes of instability-driven mass and angular momentum transport, their simplicity makes them invaluable tools for both semi-analytic modeling and simulations of long-term evolution where two- or three-dimensional calculations are too computationally costly. Despite the utility of these models, the only publicly-available frameworks for simulating them are rather specialized and non-general. Here we describe a highly flexible, general numerical method for simulating viscous thin disks with arbitrary rotation curves, viscosities, boundary conditions, grid spacings, equations of state, and rates of gain or loss of mass (e.g., through winds) and energy (e.g., through radiation). Our method is based on a conservative, finite-volume, second-order accurate discretization of the equations, which we solve using an unconditionally-stable implicit scheme. We implement Anderson acceleration to speed convergence of the scheme, and show that this leads to factor of ∼5 speed gains over non-accelerated methods in realistic problems, though the amount of speedup is highly problem-dependent. We have implemented our method in the new code Viscous Accretion Disk Evolution Resource (VADER), which is freely available for download from https://bitbucket.org/krumholz/vader/ under the terms of the GNU General Public License.

1. Introduction

Accretion disks are ubiquitous in astrophysics, in fields ranging from star and planet formation to high energy astrophysics to galaxies, and an enormous amount of effort has been invested in modeling them (e.g. Pringle, 1981). One approach to constructing such models is to conduct full two- or three-dimensional simulations, and this method offers the highest fidelity to the actual physical processes taking place in disks. However, such simulations are impractically computationally expensive for phenomena that take place over very large numbers of orbital timescales, or for disks where the characteristic scales that must be resolved for the simulation to converge are vastly smaller than the disk radial extent. For example, a long-term two-dimensional simulation of a protoplanetary disk might cover several thousand orbits, but the timescale over which planets form is millions of orbits. Quasi-periodic oscillations from disks around black holes, neutron stars, and white dwarfs can take place over similarly large numbers of orbits. In galaxies, the number of orbits is relatively modest, but the characteristic size scale of gravitational instability for the coldest phase of the interstellar medium is $\sim 10^6$ times smaller than the disk radial extent. None of these problems are amenable to solution by two- or three-dimensional simulations, at least not without extensive use of sub-grid models to ease the resolution requirements.

In such cases, one-dimensional simulations in which the disk is treated as vertically thin and axisymmetric are a standard modeling tool. The general approach in such simulations is to approximate the turbulence responsible for transporting mass and angular momentum through the disk as a viscosity, and to develop an analytic or semi-analytic model for this transport mechanism. Cast in this form, the evolution of a disk is described by a pair of one-dimensional parabolic partial differential equations for the transport of mass and energy; the form of these equations is analogous to a diffusion equation in cylindrical coordinates. Depending on the nature of the problem, these equations may have source or sink terms, may have a wide range of boundary conditions, and may have multiple sources of non-linearity.

Thus far in the astrophysical community most viscous disk evolution codes have been single-purpose, intended for particular physical regimes and modeling physical processes relevant to that regime. Thus for example there are codes intended for

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protoplanetary disks that include models for accretion onto the disk during ongoing collapse (e.g. Hueso and Guillot, 2005; Visser and Dullemond, 2010; Lyra et al., 2010; Horn et al., 2012; Benz et al., 2014), galaxy disk codes containing prescriptions for star formation (e.g. Forbes et al., 2012, 2014), and codes for simulating accretion onto compact objects that include models for magnetically-dominated coronae and have equations of state that include the radiation pressure-dominated regime (e.g. Liu et al., 2002; Mayer and Pringle, 2007; Cambier and Smith, 2013). While these codes are specialized to their particular problems, they are often solving very similar systems of equations, and thus there is a great deal of replication of effort in every community developing its own code.

This is particularly true because most of the codes are not open source, and for the most part the authors have not published detailed descriptions of their methodologies, forcing others to invent or re-discover their own. The sole exceptions of which we are aware are the GIDGET code for simulating galaxy disk evolution (Forbes et al., 2012) and the α-disk code published by Lyra et al. (2010) and Horn et al. (2012), based on the PENCIL code (Brandenburg and Dobler, 2002). Neither GIDGET nor Lyra's code are suited for general use. For example, neither allows a wide range of viscosities, equations of state, and rotation curves. Ironically, this situation is in sharp contrast to the situation for two-dimensional disk simulations, where there are a number of open source codes that include viscosity and various other physical processes. These include ZEUS-2D (Stone and Norman, 1992a,b; Stone et al., 1992), VHD (McKinney and Gammie, 2002, 2013), and PLUTO (Mignone et al., 2007). However, while it is possible to run all these codes in either a one-dimensional or pseudo-one-dimensional mode, they are all based on explicit schemes, which limits their ability to simulate very long time scales.

The goal of this paper is to introduce a very general method for computing the time evolution of viscous, thin, axisymmetric disks in one dimension, using a method suitable for simulating disks over many viscous evolution times at modest computational cost. We embed this method in a code called Viscous Accretion Disk Evolution Resource (VADER), which we have released under the GNU General Public License. VADER is available for download from https://bitbucket.org/krumholz/vader/. The code is highly flexible and modular, and allows users to specify arbitrary rotation curves, equations of state, prescriptions for the viscosity, grid geometries, boundary conditions, and source terms for both mass and energy. The equations are written in conservation form, and the resulting algorithm conserves mass, momentum, angular momentum, and energy to machine precision, as is highly desirable for simulations of very long term evolution. We employ an implicit numerical method that is unconditionally stable, allows very large time steps, and is fast thanks to modern convergence acceleration techniques. VADER is descended from GIDGET (Forbes et al., 2012, 2014) in a very general sense, but it is designed to be much more flexible and modular, while omitting many of the features (e.g., cosmological accretion and the dynamics of collisionless stars) that are specific to the problem of galaxy formation. It is implemented in C and Python. The present version is written for single processors, but we plan to develop a threaded version in the future once an open source, threaded tridiagonal matrix solver becomes available.

The plan for the remaining part of this paper is as follows. In Section 2 we introduce the underlying equations that VADER solves, and describe our algorithm for solving them. In Section 3 we present a number of tests of the code's accuracy and convergence characteristics. Section 4 discusses the efficiency and performance of the algorithm. Finally we summarize in Section 5.

2. Equations and simulation algorithm

2.1. Equations

The physical system that VADER models is a thin, axisymmetric disk of material in a time-steady gravitational potential. We consider such a disk centered at the origin and lying in the $z = 0$ plane of a cylindrical (r, ϕ, z) coordinate system. The equations of continuity and total energy conservation for such a system, written in conservation form, are (e.g., equations 1 and A13 of Krumholz and Burkert 2010)

$$\frac{\partial}{\partial t}\Sigma + \frac{1}{r}\frac{\partial}{\partial r}\left(r v_r \Sigma\right) = \dot{\Sigma}_{\rm src} \qquad (1)$$

$$\frac{\partial}{\partial t}E + \frac{1}{r}\frac{\partial}{\partial r}\left[r v_r \left(E+P\right)\right] - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{v_\phi \mathcal{T}}{2\pi r^2}\right) = \dot{E}_{\rm src}. \qquad (2)$$

Here Σ is the mass surface density in the disk,

$$E = \Sigma\left(\frac{v_\phi^2}{2} + \psi\right) + E_{\rm int} \equiv \Sigma\psi_{\rm eff} + E_{\rm int} \qquad (3)$$

is the total energy per unit area, v_ϕ is the rotation speed as a function of radius, ψ is the gravitational potential (which is related to v_ϕ by $\partial\psi/\partial r = v_\phi^2/r$), $\psi_{\rm eff} = \psi + v_\phi^2/2$ is the gravitational plus orbital energy per unit mass, $E_{\rm int}$ is the internal energy per unit area, P is the vertically-integrated pressure ($\int_{-\infty}^{\infty} p\, dz$, where p is the pressure), v_r is the radial velocity, and $\mathcal{T}$ is the torque applied by a ring of material at radius r to the adjacent ring at $r + dr$. The source terms $\dot{\Sigma}_{\rm src}$ and $\dot{E}_{\rm src}$ represent changes in the local mass and energy per unit area due to vertical transport of mass (e.g., accretion from above, mass loss due to winds, or transformation of gas into collisionless stars) or energy (e.g., radiative heating or cooling). VADER allows very general equations of state; the vertically-integrated pressure P may be an arbitrary function of r, Σ, and $E_{\rm int}$ (but not an explicit function of time).

The torque and radial velocity are related via angular momentum conservation, which implies

$$v_r = \frac{1}{2\pi r\Sigma v_\phi(1+\beta)}\frac{\partial}{\partial r}\mathcal{T}, \qquad (4)$$

where $\beta = \partial \ln v_\phi/\partial \ln r$ is the index of the rotation curve at radius r. To proceed further we require a closure relation for the torque $\mathcal{T}$. For the purposes of this calculation we adopt the Shakura and Sunyaev (1973) parameterization of this relation, slightly modified as proposed by Shu (1992). In this parameterization, the viscosity is described by a dimensionless parameter α such that

$$\mathcal{T} = -2\pi r^2\alpha(1-\beta)P. \qquad (5)$$

Note that inclusion of the $1 - \beta$ term is the modification proposed by Shu (1992), and simply serves to ensure that the torque remains proportional to the local rate of shear in a disk with constant α but non-constant β. With this definition of α, the kinematic viscosity is

$$\nu = \alpha\left(\frac{r}{v_\phi}\right)\left(\frac{P}{\Sigma}\right). \qquad (6)$$

The dimensionless viscosity α (as well as the source terms $\dot{\Sigma}_{\rm src}$ and $\dot{E}_{\rm src}$) can vary arbitrarily with position, time, Σ, and E.

Since these equations are derived in Krumholz and Burkert (2010), we will not re-derive them here, but we will pause to comment on the assumptions that underlie them, and the potential limitations those assumptions imply. The system of equations is appropriate for a slowly-evolving thin disk with negligible radial pressure support. Specifically, we assume that (1) the scale height