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Short communication

Optimal control algorithms for switched Boolean network

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Abstract

This paper investigates the time optimal control and optimal infinite-horizon control for a switched Boolean network. First, the switched Boolean network can be converted into a discrete switched system by using the semi-tensor product of matrices. Second, algorithms for time optimal control and optimal infinite-horizon control of the switched Boolean network are presented. Moreover, constrained optimal infinite-horizon control is studied. Finally, illustrative examples are given to show the efficiency of the obtained results.

1. Introduction

Boolean network was first introduced by Kauffman for modeling genetic regulatory networks [1]. In Boolean network, 0 (or 1) corresponds to the inactive (or active) state of the gene. Up to now, many results on the topological structure of a Boolean network have been presented [2–4].

Boolean control network is a Boolean network with (binary) inputs. For example, a binary input may represent whether a certain medicine is administered or not at each time step. Recently, the semi-tensor product of matrices has been successfully applied to Boolean control network. The controllability and observability of Boolean control network have been studied in

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[5]. Cheng et al. [6] have investigated the stability and stabilization of Boolean control networks. The semi-tensor product method has been used in the study of the optimal control of Boolean control network too. For example, by using a maximum principle, [7] has discussed the Mayer-type optimal control problem. The optimal infinite-horizon control of k-valued logical network and mix-valued logical network has been studied in [8,9] respectively. There have been many other results about the semi-tensor product for the multi-valued logical network, probabilistic Boolean network, etc. [10–12].

It is well known that switched systems are very important in control theory. Many biological systems appear with different model structures according to the environment changes. For example, at the inter-cellular level, cell differentiation can be described as a switched system [13]. In addition to naturally occurred switchings, switched dynamics can be the result of external intervention that attempts to re-engineer a given network by turning on and off. Hence, it is natural to present a switched Boolean network where its dynamic is governed by different switching Boolean networks. Recently, a switched Boolean network is proposed [14]. In [14], an SBCN model is constructed for the acute myeloid leukemia (AML) signaling network. However, there are few results on the control problems of switched Boolean network.

Optimal control is a fundamental concept in control theory [15–17]. Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. Time optimal control is to transfer a system from a given initial state to a special final state in minimum time. Optimal infinite-horizon control has drawn a great deal of attention too. But there have been no results on the time optimal control and optimal infinite-horizon control of a switched Boolean network to the best of our knowledge.

Motivated by the above, in this paper, based on a Floyd-like algorithm we study the time optimal control and optimal infinite-horizon control problem of a switched Boolean network. Our objective is to find a control policy which minimizes the cost function $J(u)=\overline{\lim}_{T\to\infty}(1/T)\sum_{t=1}^{T}P(x(t),u(t),\sigma(t))$. Moreover, we also investigate the constrained optimal infinite-horizon control problem. That is to minimize the above function while avoiding certain forbidden states. This seems relevant to biological systems as some states may correspond to unfavorable or dangerous situations.

The rest of this work is organized as follows. Section 2 gives the preliminaries. Then based on the Floyd-like algorithm proposed by [9], an algorithm for time optimal control is presented. Finally, algorithms are given to find the optimal cycle in the unconstrained and constrained cases. The optimal control and switching design can be obtained too. Illustrative examples are given to support our results which are followed by the conclusion in Section 5.

2. Preliminaries

In this paper, the matrix product we use is the semi-tensor product (STP).

Definition 2.1 (*Cheng et al. [18]*). For $M\in\mathbb{R}^{m\times n}$ and $N\in\mathbb{R}^{p\times q}$, their STP (also called Cheng product), denoted by $M\ltimes N$, is defined as follows:

$$M\ltimes N := (M\otimes I_{s/n})(N\otimes I_{s/p}),$$

where s is the least common multiple of n and p and $\otimes$ is the Kronecker product.

STP is a generalization of conventional matrix product. All the fundamental properties of conventional matrix product remain true. Based on this, we can omit the symbol $\ltimes$. There are also some basic properties of STP, for details, see [19].