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On semi-global stabilization of linear periodic systems with control magnitude and energy saturations

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Abstract

This paper establishes a systematic approach to solve the $L_\infty(l_\infty)$ and $L_2(l_2)$ semi-global stabilization problem of linear periodic systems with controls having bounded magnitude and energy, respectively. The developed approach will be referred to as $L_\infty(l_\infty)$ and $L_2(l_2)$ low gain feedback. Definitions, properties, and characterizations of this new concept are also provided, and particularly, the characterizations are based upon differential (difference) Lyapunov inequalities. Design of $L_\infty(l_\infty)$ and $L_2(l_2)$ low gain feedback by solving differential (difference) Riccati equations is proposed. Both continuous-time and discrete-time linear periodic systems are studied and both state feedback and observer based output feedback are considered. In the discrete-time setting, a linear matrix inequalities (LMIs) based solution to the l_∞ and l_2 semi-global stabilization problem is also established and the LMIs conditions are shown to be always solvable. Applications of the proposed approach to the elliptical spacecraft rendezvous system show the effectiveness of the established theory.

1. Introduction

Control of time-varying systems has received much attention in the past several decades and many results have been published in the literature (see [31] and the references therein). Periodic systems as special cases of time-varying systems have also been extensively studied in the literature (see [2,3,7,10,16,17,32] and the references given there). It has been clear that linear

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periodic systems possess many important properties belonging to linear time-invariant systems because of the well-known Floquet–Lyapunov theory [1]. Also, it is clear that continuous-time linear periodic systems are quite different from discrete-time linear periodic systems since the latter ones can be reformulated as time-invariant systems by using some lifting techniques (see, for example, [1] and [15]). Hence, it is believed that control problems, if not all, for continuous-time linear periodic systems are harder to solve than discrete-time linear periodic systems [28]. On the other hand, saturation nonlinearity exists in every practical control systems and the ignorance of such a nonlinearity will lead to performance degradation and even instability of the practical control system [8]. Hence control systems design by taking the actuator saturation nonlinearity into consideration has received much attention during the past several decades and a great number of results have been published in the literature (see [8,11,18,19,27,29,30] and the references therein).

The design of periodic control systems with controls having either bounded magnitude or bounded energy is important since it has some potential applications in engineering. For example, the spacecraft rendezvous problem belongs to this type since the linearized Tschauner–Hempel equations characterizing the relative motion is a continuous-time linear periodic system and the control accelerations being the control of the system are always bounded in both the L_∞ and L_2 sense [26]. For the constrained control problems of linear periodic systems, some results are already available in the literature. For example, a discrete-time linear plant that is asymptotically null controllable by bounded controls is semi-globally stabilized by bounded periodic feedback via a lifted representation of the time-invariant plant in [6]; a polynomial approach is developed in [5] to solve the local stabilization problem of an SISO discrete-time linear periodic system with bounded controls; local stability analysis and stabilization of discrete-time linear periodic systems with bounded controls are considered in [24] by using the concept of periodic invariant sets, semi-global stabilization of discrete-time and continuous-time linear periodic systems are respectively studied in [25,33] and [28] by using periodic Lyapunov differential and difference equations, and the global stabilization of neutrally stable linear periodic systems by bounded feedback was solved very recently in [34] by linear feedback.

By considering that the solutions to the semi-global stabilization problems of linear periodic systems established in our recent work [25,33,28] are constructive, this paper will establish a *systematic* approach for solving $L_\infty(l_\infty)$ and $L_2(l_2)$ semi-global stabilization problems for linear periodic systems with controls having bounded magnitude and bounded energy, respectively. The established approaches will be referred to as periodic low gain feedback. Characterizations of periodic low gain feedback in terms of Lyapunov differential and difference inequalities are established. Analytical methods based on the periodic differential Riccati equation (DREs) and periodic difference Riccati equations (DcREs) will be developed. These results generalize our recent work on linear time-invariant systems in [27,29] to the periodic setting. For the l_∞ and l_2 semi-global stabilization of discrete-time linear periodic systems, a computational approach based on linear matrix inequalities (LMIs) is also established. An application of the proposed approach to the elliptical spacecraft rendezvous system will be carried out to illustrate the effectiveness of the proposed approach.

The remainder of this paper is organized as follows. Semi-global stabilization of continuous-time and discrete-time linear periodic systems are respectively studied in Sections 2 and 3 and applications of the developed theory to the elliptical spacecraft rendezvous system will be investigated in Section 4. Finally, Section 5 concludes this paper.

Notation: Throughout this paper, we will use standard notation. For a matrix A, we use $|A|$ and $|A|_\infty$ to denote its 2 and ∞ norms and we use $\lambda(A), A^{\mathrm{T}}$, and $\mathrm{tr}(A)$ to denote its eigenvalue set, its