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# Optimal switching between autonomous subsystems

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#### Abstract

A novel scheme is presented for solving the problem of optimal switching with nonlinear autonomous subsystems. This scheme determines the approximate global optimal solution for different initial conditions in a feedback form. Restrictions, including the need to enforce a mode sequence and/or a number of switching, do not exist for the developed method. Performance is evaluated in several examples with different complexities and the numerical simulation shows great promises for the controller.

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#### 1. Introduction

Optimal scheduling of systems with a switching nature has attracted many researchers during the last decade [1–21]. A switching system is comprised of subsystems with different dynamics which at each time instant only one of them is active. Hence, controlling these systems includes determining both 'when' to switch and 'what mode' to switch to. Systems with such a nature appear in different fields, from trajectory planning to disease therapy [1–5].

The developments in the field of optimal switching can be divided to two main categories: nonlinear programming based developments and discretization based developments. The former utilizes the gradient of the cost with respect to the switching instants to calculate the local optimal switching times [6–13]. In these developments, the sequence of active subsystems, known as mode sequence, is typically selected a priori. The problem is determining the switching instants between the modes. Among nonlinear programming based methods, some ideas are presented in [13,14] for admitting free mode sequence conditions. In [13], a two stage optimization algorithm

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was developed where in one stage the switching time is updated and at another stage the mode sequence is modified. In [14], this process was improved such that a single stage algorithm which solely updates the mode sequence for the selected initial condition is utilized.

Discretization based developments include studies that discretize the switching problem to deal with a *finite* number of options. An optimization scheme was developed in [1] to find both the optimal mode sequence and the switching time for positive linear systems. A direct search has been utilized in [15] to evaluate the cost function for different randomly selected switching time sequences. The discretization of both the state and input spaces was used to calculate the value function for optimal switching through dynamic programming in [16]. Genetic algorithm and neural networks were used in [17] and [18], respectively, to determine the optimal switching for a preselected initial condition within intelligent methods.

Each of these methods requires a large amount of computations to numerically find the optimal switching time for an a priori selected initial condition. Each time the initial condition is changed, a new set of computations must be performed to find the corresponding optimal switching instants. In [9] the validity of the results was extended for different initial conditions within a pre-selected set. This is done through determining the switching parameter as the local optimum in the sense that it minimizes the worst possible cost for all trajectories starting in the selected set of initial states. A neural network based method for optimal switching was recently proposed in [19] by the authors of this study for problems with *fixed* mode sequence. Once the network is trained based on the algorithm given in [19], the optimal switching scheme for every selected initial condition can be calculated through a static function minimization for online implementation. Some other researchers have focused on stabilization of switching systems, for example [20,21]. Interested readers are referred to [22] for a theoretical analysis on the properties of the value function for discrete-time switching systems with linear dynamics and quadratic cost function terms.

The contribution of this work is developing a scheme to solve the optimal switching problem for systems with nonlinear autonomous subsystems. The only control to be determined, in switching problems with autonomous subsystems, is the active mode at each instant. A number of papers, including [1,4,5,8-10,13,14], focus on such problems. The scheme presented in this study is based on the Bellman principle of optimality [23], providing optimal solution in realtime. To this goal, the function representing the nonlinear mapping between the optimal cost-togo as the output, and the current state and time as the inputs, is required. An algorithm is developed, motivated by studies in adaptive critics (AC) [24-26], to learn this function with a desired degree of accuracy. This function approximation is done through utilizing a neural network (NN) as a function approximator and training it using the algorithm. The developed method has the following four advantages differentiating it from methods available in the literature: (1) This method offers approximate global optimal switching instants versus local optimal ones resulting from nonlinear programming based methods. (2) This method does not require enforcing a mode sequence or a number of switching. (3) The solution is calculated in a feedback form. Hence, it will have the relative robustness of feedback controllers toward unmodeled disturbances, compared with open loop solutions. (4) This method offers an optimal solution for a vast domain of initial conditions. Thus, an optimal switching solution for different initial conditions can be readily calculated using the same trained NN.

This article is organized as follows. Problem formulation is presented in section II. The main idea of the proposed method is given in section III. The process of approximating the cost-to-go function is discussed in section IV. Details regarding the implementation of the method for online control are presented in section V. Simulation studies are given in section VI, followed by concluding remarks.

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