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# Globally sparse and locally dense signal recovery for compressed sensing

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**Abstract**

Sparsity regularized least squares are very popular for the solution of the underdetermined linear inverse problem. One of the recent progress is that structural information is incorporated to the sparse signal recovery for compressed sensing. Sparse group signal model, which is also called block-sparse signal, is one example in this way. In this paper, the internal structure of each group is further defined to get the globally sparse and locally dense group signal model. It assumes that most of the entries in the active groups are nonzero. To estimate this newly defined signal, minimization of the $\ell_1$ norm of the total variation is incorporated to the group Lasso which is the combination of a sparsity constraint and a data fitting constraint. The newly proposed optimization model is called globally sparse and locally dense group Lasso. The added total variation based constraint can encourage local dense distribution in each group. Theoretical analysis is performed to give a class of theoretical sufficient conditions to guarantee successful recovery. Simulations demonstrate the proposed method's performance gains against Lasso and group Lasso.


## 1. Introduction

Compressive sensing (CS) is a new signal processing technique that can reconstruct the signal with a much fewer randomized samples than Nyquist sampling with high probability on condition that the signal has a sparse representation. Least-absolute shrinkage and selection operator (Lasso) is one of the popular convex ways to exploiting sparsity to recover the sparse signal [1]. It has wide applications in signal processing [2], machine learning [1], etc. Recent research on sparse signal recovery for CS shows that adding structural constraint can improve the estimate performance [3].

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Group Lasso (GLasso) is a popular example of model based sparse signal recovery ways [4–6], which extends the popular Lasso method [1,2]. Besides sparsity, it takes another physical property of signal. Group-sparsity is called block-sparsity too [7–9]. In group-sparse signal model, most of the nonzero entries are accumulated in groups. This means that a few groups can represent the signal. The GLasso replaces the $\ell_1$ norm regularization with a sum of $\ell_2$ norm regularization which has the effect of grouping all the variables within each group and the recovery entries are encouraged to be zero or nonzero simultaneously. A series of literatures has addressed certain statistical properties of the GLasso. In [6,10,11], some asymptotic properties like risk consistency and estimation consistency are provided. Some sufficient conditions for successful recovery are given in [7,8,12]. To allow overlapping of different groups, overlapping Group Lasso is proposed and corresponding algorithms are developed [13,14]. Weighted Group Lasso was proposed to enhance the sparsity [15–18]. Combining the $\ell_1$ norm regularization for standard sparse constraint and the sum of $\ell_2$ norm regularization for group constraint, the Hi-Lasso was proposed to further enforce sparse distribution in each group [19–24]. There are a lot of papers on Group Lasso, we just name parts of them here.

In this paper, internal structural information of active groups is used to set a globally sparse and locally dense group signal model, and it is different from the signal model for the HiLasso. In the proposed signal model, the estimated signal is globally sparse and locally dense. This kind of signal model can be applied to many areas, such as wireless communication [18], array signal processing [17] regression [5] and bioinformatics [13]. A corresponding novel convex recovery model called globally sparse and locally dense (SDGLasso) is developed with a new constraint added for the signal recovery. This newly added constraint, in the form of total variation minimization, encourages dense distribution in each group. Similar to the Lasso [25,26], the GLasso [8] and the HiLasso [21], coherence evaluation for the proposed SDGLasso is performed to get a group of sufficient conditions for the successful recovery of globally sparse and locally dense group signal. Numerical experiments show the performance improvement.

In the rest of this paper, the globally sparse and locally dense group signal model is formulated in Section 2. And in Section 3, a corresponding recovery model called SDGLasso is proposed. Section 4 gives a class of sufficient conditions for the SDGLasso and its deduction. In Section 5, simulations are performed to demonstrate the performance improvement of the proposed method. Finally, Section 6 draws the conclusion.

## 2. Signal model

Considering an $N$-by-1 signal $\mathbf{x}$ can be expanded in an $N$-by-$N$ orthogonal complete dictionary $\mathbf{\Psi}$, with the representation as

$$\mathbf{x} = \mathbf{\Psi}\boldsymbol{\theta} \tag{1}$$

When most elements of the $N$-by-1 vector $\boldsymbol{\theta}$ are zeros or trivially small, the signal $\mathbf{x}$ is sparse. When the number of nonzero entries of $\boldsymbol{\theta}$ is $S$ ($S \ll N$), the signal is said to be $S$-sparse.

The signal model (1) is the standard sparse one. In many applications, practical signals enjoy some other common structural information. One example is the sparse group signal model [7]

$$\boldsymbol{\theta}^T = \left[ \underbrace{\theta_1 \cdots \theta_d}_{\boldsymbol{\theta}_1^T} \ \underbrace{\theta_{d+1} \cdots \theta_{2d}}_{\boldsymbol{\theta}_2^T} \ \cdots \ \underbrace{\theta_{(K-1)d+1} \cdots \theta_N}_{\boldsymbol{\theta}_K^T} \right] \tag{2}$$