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Towards a port-based formulation of macro-economic systems

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Abstract

This paper aims at extending the port-Hamiltonian approach to a simple class of macro-economic systems by considering the dynamics as the result of the interaction between a limited set of "atomic components," e.g. inventories, (re)investments, suppliers and demand. Once flow, effort, and "power" (i.e., the cash flow) have been defined, the behaviour of these simple elements is provided, and their interconnection is described in terms of Dirac and contact structures. The first ones, associated to power conservation in physical modelling, correspond to the law of good bookkeeping (Walras's Law), in economy. Differently, contact structures appear in the description of irreversible phenomena, and in macro-economy they are employed in a more realistic formulation of the firm (or supplier) behaviour. The firm, in fact, is a profit maximising entity that interconnects the markets associated to the demand and to the input factors in order to create a profit. Several examples are provided to show to effectiveness of the approach.

1. Introduction

The port-Hamiltonian approach is a systematic framework for geometric modelling and control of lumped-parameter physical systems [1,2], and it is a mathematical formalisation of bond-graphs [3]. In this respect, a physical system is the result of a network of atomic multi-port elements, each characterised by a particular energetic behaviour. The key point is the identification of the interconnection structure, mathematically described by a Dirac structure [4],

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generalisation of the Kirchhoff laws, that describes the power flows among different parts of the whole system, and between the system and the environment. This approach can be fruitfully applied for modelling a wide class of physical (mechanical, electrical, hydraulic and chemical) systems, and several control techniques, based on energy considerations, have been developed in order to solve the regulation problem [2,5]. On the other hand, at the best of the author's knowledge, no attempt has been made in trying to extend the port-Hamiltonian formalism beyond physical systems, e.g. to (macro-)economic systems, as proposed in this paper.

There are several systematic ways to describe the dynamics of economic systems via interconnection of simpler sub-systems: an example is represented by *system dynamics* [6,7] where, similar to bond-graphs, a limited set of components (e.g., causal loops, stocks and flows) are properly interconnected to have visual and mathematical descriptions of the system evolution. However, such components are not explicitly characterised in terms of their energetic properties. Differently, it is worth noting that there have been some attempts to extend the bond-graph formalism to macro-economic systems, and these works represent the starting point of this paper. In fact, in [8–10], an effective way to define the space of power variables (i.e., flow of orders and unitary cost of a product), the associated power (i.e., the cash flow), and the energy-storage (i.e., inventories and (re)investment), and dissipative elements (i.e., supply and demand) is discussed. In this paper, by formalising the network structure that drives the money flow between each sub-system in terms of a Dirac structure, the port-Hamiltonian and the associated Brayton–Moser [2,11,12] formulations of the dynamic are obtained. On the other hand, as discussed in [13], the use of power-conserving interconnections (Dirac structures), as the unique way to describe the flow of money, has some intrinsic limitations, in particular when an accurate representation of the *firm*, or *supplier*, is needed.

Within the port-based framework discussed here, a firm realises a particular interconnection between demand and input factor markets, that is meant to maximise the profit under the constraint that the quantity of produced goods is a well-defined function of the different inputs. Such inputs are provided e.g. by the financial, labour, and raw material markets, and the function that establishes this input/output connection is the *production function*. Typical examples are the Cobb–Douglas or the C.E.S. production functions [14]. This “component” is responsible for a sort of “irreversible process” associated with the profit creation. It is not surprising to discover here a strong relation between economic and thermodynamic systems. Within the port-Hamiltonian formalism, it has been already shown that Dirac structures are not the most appropriate tool for the description of the “topology” of thermodynamic systems, and that it is more convenient to rely on contact forms [15], and on port-contact systems [16,17]. This is in line with several works aiming at showing that the *geometry* behind a production function admits a description in terms of contact forms, see e.g. [18] and references therein. In this paper, starting from the associated production function, the supplier description in terms of a contact structure is derived. Such contact structure is then employed to interconnect a set of port-Hamiltonian systems modelling all the different markets that the firm is going to relate.

The paper is organised as follows. In Section 2, a background on contact and Dirac structures, and on port-Hamiltonian systems is provided. Flow and effort variables, and associated power are defined for macro-economic systems in Section 3, the main atomic elements are introduced in Section 4, and the port-Hamiltonian and Brayton–Moser formulations are obtained in Section 5. The supplier behaviour is discussed in Section 6, where the port-Hamiltonian description previously introduced is combined with contact structures, and a general framework for macro-economic system modelling based on “port” and “interconnection” concepts is obtained. Finally, conclusions are in Section 7.

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