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Sliding mode mean square filter design for linear stochastic time delay systems☆

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Abstract

This paper investigates the mean-square filtering problem for a linear time delay systems with Gaussian white noises. The obtained solution contains a sliding mode term, signum of the innovations process. It is demonstrated that the estimate produced by the designed filter generates the mean-square estimate, which has the same minimum estimation-error variance as the best estimate given by the classical Kalman–Bucy filter. The theoretical result is applied to an illustrative example: the tryptophan operon of *E. coli*, verifying the performance of the designed filter. It is demonstrated that the estimates produced by the designed sliding-mode mean-square filter and the Kalman–Bucy filter yield the same estimation-error variance. Simulation graphs demonstrate the better performance of the designed sliding-mode filter and show the potential of the proposed new filter.

1. Introduction

Sliding mode theory was developed in the early 1970 (see a historical review in [1–3]), it has been applied to solve several classes of problems. For instance, the sliding mode control methodology has been used in tracking [4,5], stabilization [6,7], frequency domain analysis [8], observer design [9,10], and other control problems. Promising modifications of the original

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sliding mode concept, such as higher order sliding modes [11,3] and integral sliding mode [12], have been developed. Applications of the sliding mode method were extended even to stochastic systems [13–17] and stochastic filtering problems [18–21,34–36]. The sliding mode technique has also being extensively used for industrial applications [22–27]. However, to the best of authors' knowledge, it does not exist as a sliding mode filtering algorithms that solve the mean-square filtering problems for linear state time delay systems.

On the other hand, time delays or lags are common in chemical engineering processes and many physical and biological systems. They give rise to complex dynamic phenomena [28–30]. In biology, the delays can represent gestation times, incubation periods, transport delays, or can simply lump complicated biological processes together, accounting only for the time required for these processes to occur. In the context of tumor growth, transcriptional regulator dynamics and genetic network time delays are present too. Furthermore, nonlinearity and noise are fundamental properties of biological systems [31–33].

This paper presents an alternative solution to the mean-square filtering problem for linear state time delay systems, which contains a sliding mode term, signum of the innovations process. To the best of our knowledge, this is the first designed sliding mode filter that is optimal with respect to the mean-square criterion, and it produced the estimate with the same structural properties as the conventional optimal filter for linear state time delay systems. To illustrate the effectiveness of the filter designed, it was applied to the tryptophan operon of *E. coli*, and our simulations results were compared with the estimates produced by the Kalman–Bucy filter. This algorithm produced a best estimate when exist time delays in the system.

It should be pointed out that the designed filter present a considerable improvement in the filtering theory and its applications. Since the obtained filtering equations have a simpler structure even in comparison to the classical Kalman–Bucy filter. This enables to make more effective industrial applications of the mean-square filter to mechanical, power, robotic, biology and other technical plants [37–42], where state estimation problem should be solved in the presence of unbounded stochastic noises. The simpler filter structure is achieved in view of the design of the innovations process as a signum function, in contrast to the filters designed in [18–21,43].

This paper is organized as follows. The mean-square filtering problem for linear state time delay systems with Gaussian white noise is described in Section 2. The sliding mode solution to the mean-square filtering problem is obtained in Section 3. Furthermore, it contains an illustrative example to the tryptophan operon of *E. coli*, and Section 4 gives the general conclusions to this paper.

2. Problem statement

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with an increasing right-continuous family of σ-algebras $\mathcal{F}_t, t \geq t_0$, and let $(\omega_1(t), \mathcal{F}_t, t \geq t_0)$ and $(\omega_2(t), \mathcal{F}_t, t \geq t_0)$ be independent standard Wiener processes. The $\mathcal{F}_t-$measurable random process $(z(t), y(t))$ is described by a linear differential equation for the dynamic system state:

$$dz(t) = (\alpha_0(t)z(t) + \alpha_1(t)z(t-h))\,dt + \gamma(t)\,d\omega_1(t), \quad z(t_0) = z_0, \tag{1}$$

with the initial condition: $z(s) = \phi(s), s \in [t_0 - h, t_0]$, and a linear differential equation for the observation process:

$$dY(t) = \beta(t)z(t)\,dt + \Gamma(t)\,d\omega_2(t). \tag{2}$$