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Adaptive unit-vector control of an uncertain heat diffusion process

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Abstract

This paper deals with the distributed tracking control of a heat process having uncertain diffusivity and subject to a distributed disturbance whose L_2 norm is bounded by a constant which is not known a priori. Under certain regularity assumptions on the disturbance and on the chosen reference profile, a distributed unit-vector control, with an adaptive magnitude, is designed which provides the asymptotic tracking of the reference. The logic governing the gain adaptation is gradient-based and monodirectional, i.e. the gain cannot decrease over time. Lyapunov arguments are invoked to support the convergence properties of the proposed scheme, whose performance are also investigated by means of computer simulations.

1. Introduction

Sliding-mode control has long been recognized as a powerful control method to counteract nonvanishing external disturbances affecting the dynamics of the plant to be controlled [31]. This method is based on enforcing sliding motions into the control system, and, since the motion along the sliding manifold proves to be uncorrupted by matched disturbances, the closed-loop system is guaranteed to exhibit strong properties of robustness against significant classes of disturbances and model uncertainties. Due to these advantages and simplicity of implementation, sliding-mode controllers have widely been used in various applications [26].

Many important systems and industrial processes are governed by partial differential equations and are often described by models with a significant degree of uncertainty. Therefore, research interest has emerged in extending the discontinuous control methods to

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infinite-dimensional systems. Presently, the discontinuous control synthesis in the infinite-dimensional setting is well documented [17,7] and it is generally shown to retain the main robustness features as those possessed by the finite-dimensional counterparts. Several robust control paradigms have been fruitfully applied in the infinite dimensional setting such as adaptive control [15,16], H_∞ and LMI-based design [8,9].

The unit feedback control of finite-dimensional systems has been shown to guarantee a certain degree of robustness with respect to uniformly bounded uncertainties [2]. First applications [5,18] of these algorithms to distributed parameter systems corroborated their utility for infinite-dimensional systems as well, and motivated further theoretical investigations [19–23].

While adaptive control of finite dimensional systems is a mature area that has produced effective methods for most LTI systems of interest [11], adaptive control techniques have been developed for only a few of the classes of PDEs for which nonadaptive controllers exist. The existing results [13,30,3,4,10] focus on model reference (MRAC) type schemes and the control action distributed in the PDE domain (see [15] for a more detailed literature review). One of the major obstacles in developing adaptive schemes for PDEs is the absence of parameterized families of stabilizing controllers. In [28,14], some explicit formulae were introduced in the domain of boundary control of parabolic PDEs.

Here we deal with the state-feedback distributed tracking control of a heat process having uncertain diffusivity and subject to a distributed disturbance which is supposed just to fulfill simple norm-boundedness and smoothness requirements. As the main novelty over, e.g. [22], the disturbance L_2 norm is assumed to be bounded by a constant which is supposed not known a priori. The proposed solution consists of a distributed unit-vector control with an adaptively adjusted scalar gain parameter. The adaptation logic is gradient-based and monodirectional, i.e. the gain cannot decrease over time. The form of the proposed adaptation law shares some similarity with that in ([15], Theorem 1), however the field of application is different in that in [15], within a boundary control setup, adaptation was aimed to estimate the uncertain reaction coefficient in a reaction–diffusion equation. More close similarities can be found between the suggested adaptation and those, monodirectional and gradient based, considered in [27,6], that were, however, exclusively confined to the finite dimensional setting. In the present paper, in analogy with [27,6], adaptation is devoted to find an overestimate of a certain constant, a priori unknown, which represents an upperbound to the L_2 norm of some uncertainty term. The application of this idea in the framework of distributed parameter systems raises major challenges in developing the appropriate Lyapunov based stability analysis, and this paper overcomes these problems by means of a dedicated "two-stage" Lyapunov analysis where two different Lyapunov functions, together with the Barbalat Lemma, are invoked to prove the claim.

The rest of the paper is outlined as follows. Section 1.1 recalls some preliminary definitions and results. Section 2 outlines the main result of this paper by orderly presenting the problem formulation, in Section 2.1, and the controller synthesis, complemented by the appropriate Lyapunov based convergence proof, in Section 2.2. Section 3 discusses some simulation results and Section 4 draws some concluding remarks and perspectives for next research work.

1.1. Notation and preliminaries

The notation used throughout is fairly standard. $L_p(a,b)$, with $p \geq 1$ and $a \leq b$, stands for the Hilbert space of square integrable functions $z(\zeta)$, $\zeta \in [a,b]$, with the corresponding