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Sliding mode filtering for stochastic systems with polynomial state and observation equations

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Abstract

In this paper, the mean-square and mean-module filtering problems for polynomial system states over polynomial observations are studied proceeding from the general expression for the stochastic Ito differentials of the estimate and the error variance. The paper deals with the general case of nonlinear polynomial states and observations. As a result, the Ito differentials for the estimates and error variances corresponding to the stated filtering problems are first derived. The procedure for obtaining an approximate closed-form finite-dimensional system of the sliding mode filtering equations for any polynomial state over observations with any polynomial drift is then established. In the examples, the obtained sliding mode filters are applied to solve the third-order sensor filtering problems for a quadratic state, assuming a conditionally Gaussian initial condition for the extended second-order state vector. The simulation results show that the designed sliding mode filters yield reliable and rapidly converging estimates.

1. Introduction

Although the general solution of the mean-square filtering problem for nonlinear state and observation equations confused with white Gaussian noises is given by the Kushner equation for the conditional density of an unobserved state with respect to observations [1], there are a very few known examples of nonlinear systems where the Kushner equation

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can be reduced to a finite-dimensional closed system of filtering equations for a certain number of lower conditional moments (see [2–4] for more details). The complete classification of the "general situation" cases (this means that there are no special assumptions on the structure of state and observation equations and the initial conditions), where the nonlinear finite-dimensional filter exists, is given in [5]. There also exists an extensive bibliography on robust, in particular, H_∞ filtering for linear [6–9] and nonlinear [11–31] stochastic systems, have also been made sliding mode approach to H_∞ synchronization (see [10]). The sliding mode mean-square and mean-module filters have recently been designed for linear [25] and polynomial [26,27] systems. However, the cited papers did not consider the filtering problems for polynomial systems, where both, state and observation, equations include polynomial functions of the system state in the right-hand sides.

This paper presents an approximate sliding mode finite-dimensional mean-square and mean-module filters for polynomial system states over polynomial observations, continuing the research in the area of the sliding mode filtering, which has been initiated in [25–27]. In contrast to the previously obtained results, the paper deals with the general case of nonlinear polynomial states and observations. Designing a closed-form sliding mode finite-dimensional filter over polynomial observations presents a significant advantage in the filtering theory and practice, since it enables one to address some filtering problems with state and observation nonlinearities, such as the cubic sensor problem [35] for various polynomial systems. Indeed, the main paper result allows one to design a suboptimal sliding mode finite-dimensional filter for any polynomial state over polynomial observations. Furthermore, since any nonlinear function can be approximated by a polynomial of a certain degree up to any precision, this would potentially lead to designing a suboptimal sliding mode finite-dimensional filter for any nonlinear state over observations with a nonlinear drift. The stated sliding mode filtering problem is treated proceeding from the general expression for the stochastic Ito differentials of the mean-square and mean-module estimate and the error variance [36]. As the first result, the Ito differentials for the mean-square and mean-module estimates and error variance corresponding to the stated sliding mode filtering problem are derived. Then, a closed-form sliding mode finite-dimensional system of the filtering equations with respect to a finite number of filtering variables can be obtained for a polynomial observation equation, additionally assuming a conditionally Gaussian initial condition for the higher degree states. This assumption is quite admissible in the filtering framework, since the real distribution of the entire state vector is actually unknown. In this case, the corresponding procedure for designing the closed-form sliding mode filtering equations is suggested.

As an illustrative example, the closed system of the sliding mode filtering equations with respect to two variables, the mean-square and mean-module estimate and the error variance, is derived in the explicit form for the particular case of a quadratic state and third-order polynomial observations, assuming a conditionally Gaussian initial condition for the extended third-order state vector. This filtering problem generalizes the cubic sensor problem stated in [35]. The resulting sliding mode filters yield reliable and rapidly converging estimates, in spite of a significant difference in the initial conditions between the state and estimate, whereas the extended Kalman–Bucy filter [37] and the conventional mean-square polynomial filter [38], behaves less satisfactorily.

The paper is organized as follows. Section 2 presents the mean-square and mean-module filtering problem statements for polynomial system states over polynomial observations. The Ito differentials for the mean-square and mean-module estimate and the error variance are derived in Section 3, where the procedure for obtaining an approximate sliding mode