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Robust stabilization of nonlinear time delay systems: A complete type functionals approach

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Abstract

The robust stabilization of some classes of nonlinear delay systems with nominal linear delay system is addressed. The form of the controller is not an a priori proposal, but it is the result of a synthesis relying on the use of complete type Lyapunov–Krasovskii functionals, leading to distributed delay linear or nonlinear robust control laws. Simulation results of the stabilization of a chemical refining process demonstrate the good performance of the proposed approaches.

1. Introduction

The design of stabilizing controllers for time delay systems has been a widely studied topic in the last decades [9,21], both in the framework of the Lyapunov–Krasovskii approach [4,27,28] and the Lyapunov–Razumikhin approach [11].

Two types of Lyapunov–Krasovskii functionals are available: the prescribed or "reduced" type which are the product of the ingeniosity of the proponent, and the complete type functionals, whose form is obtained via integration of a prescribed negative derivative along the trajectories of the system [20]. The strength of this class of functionals is that, if the system is stable, their existence is guaranteed and they admit a quadratic lower bound [13]. In recent contributions these complete type functionals

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have been modified with the introduction of cross terms [16,25] or derivative terms [6] in the derivative, aiming at improving robust stability bounds of autonomous retarded linear delay systems. The synthesis of controllers based on complete type functionals has been addressed in the past: the functional in [25] was employed in the design of suboptimal controllers for stable systems with pointwise and distributed delay [23]. The one introduced in [13] was used in the Guaranteed Cost Control strategy proposed in [24], allowing structured perturbations of the system matrices. There, the particular structure of the derivative has been shown to be instrumental in the determination of the idoneous form of the control law.

The purpose of this contribution is to use this useful feature of complete type functionals to address the synthesis of robust controllers for nonlinear systems with nominal known linear delay model and nonlinear disturbances. Notice that while the linear nominal time delay system must be stable for the functional to exist, the perturbed system can be unstable.

Two cases are studied. The case of linear systems with nonlinear perturbations satisfying the matching condition and a Lipschitz-like condition is first addressed, allowing a clear exposition of the underlying proof strategy. Second, the matching condition is relaxed, and the nonlinear disturbance is assumed to belong to a given convex space; the problem is then addressed in the framework of the Control Robust Lyapunov Function approach [5] based on the universal formula introduced for nonlinear delay free systems in [5,26].

In the past, the robust stabilization of time delay nonlinear systems has been addressed, provided a Lyapunov–Razumikhin function, by using the Control Lyapunov Function methodology [10–12,29]. Results have been also obtained, in the Lyapunov–Krasovskii framework, all based on functionals of prescribed form, such as the simple form employed in [19], the sum of squares parameterized form introduced in [18] or the one introduced for robust fuzzy control in [3,30].

In contrast with these approaches, our control law is not the result of an a priori chosen functional (or function), but it is the result of using the philosophy of the Control Lyapunov Function framework [5,26], to achieve the negativity of the derivative of the complete type functional along the trajectories of the perturbed system. The resulting linear and nonlinear controllers include distributed delays, in other words, they capture all the state [14] of the system. Such a form appears to be the natural form for controllers of time delay systems, as seen for example in [23].

The contribution is organized as follows: fundamental concepts and results on complete type functionals are recalled in Section 2. Section 3 is devoted to the determination of the form of controllers for nonlinear time delay systems via the complete type functional. A system with nominal linear stable delay model and nonlinear perturbation satisfying a Lipschitz-like condition and a matching condition is analyzed in Section 3.1. In Section 3.2, as [5], we obtain for nonlinear disturbances belonging to a convex space, nonlinear robust controllers by solving a max problem. In Section 3.3, the controllers obtained when starting with the functional with derivative including cross terms introduced in [6] are given without proofs. The above results are validated with the control of the linearized model of a chemical refining process with transport lag, and with comparisons, based on simulation results, with other control strategies in Section 4. The note ends with some concluding remarks.

2. Preliminary results

Consider the single delay system of the form

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h),$$