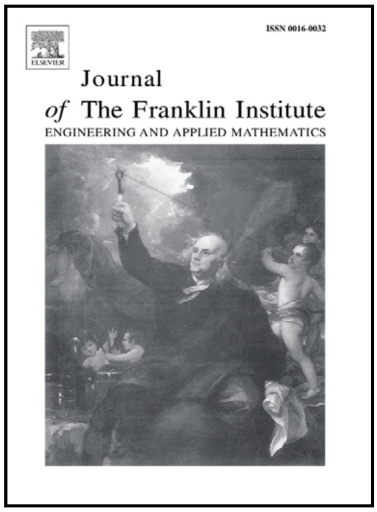

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A property of the eigenvalues of the symmetric positive definite matrix and the iterative algorithm for coupled Sylvester matrix equations

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Abstract

In this paper, we discuss the properties of the eigenvalues related to the symmetric positive definite matrices. Several new results are established to express the structures and bounds of the eigenvalues. Using these results, a family of iterative algorithms are presented for the matrix equation AX=F and the coupled Sylvester matrix equations. The analysis shows that the iterative solutions given by the least squares based iterative algorithms converge to their true values for any initial conditions. The effectiveness of the proposed iterative algorithm is illustrated by a numerical example.

Key words: Eigenvalue, Iterative algorithm, Matrix equation, Symmetric positive definite matrix

1. Introduction

Matrix equations are often encountered in the system theory, control theory and some areas of the pure and applied mathematics [1, 2, 3]. Many publications have studied how to solve different types of matrix equations, e.g., [4]. Though the formula solutions are vital in the theoretical derivation, the numerical solutions play the important roles in the practical applications [5, 6, 7]. If the dimensions of the coefficient matrices are small then the Gaussian elimination or other direct methods are effective. With the increase of the sizes of the related matrices, the iterative approaches have replaced the direct methods and become the main strategy of solving the matrix equations [8]. So the iterative algorithms for finding the numerical solutions have received mach attentions in many areas [9, 10], including signal processing and parameter estimation [11, 12, 13, 14] and system modeling [15, 16, 17, 18, 19].

Recently, by using the hierarchical identification principle, Ding and his coworkers presented several iterative schemes for solving matrix equations [20, 21, 22]. The schemes, resembling the classical Jacobi and Gaussian iterations [5, 6] for linear systems, are easy to implement and cost little per step and are convergent linearly at the best. Gradient and least squares based iterative algorithms have also been proposed for (coupled) matrix equations [23, 24], which are applicable to the Lyapunov matrix equations and Sylvester matrix equations as special cases [25]. Inspired by this work, with the real representations of complex matrices, Wu et al. presented the gradient based iterative algorithms for solving the complex (coupled) matrix equations [26]; Song et al. proposed the gradient based iterative algorithms for the generalized coupled conjugate transpose matrix equations [27].

Contrasting to the flexibility and approval of the gradient based iterative algorithms [28], the least squares based iterative algorithms encounter obstacles in deriving the iterative algorithms for solving the matrix equations $\boldsymbol{AX}+\boldsymbol{X}^{\mathrm{T}}\boldsymbol{B}=\boldsymbol{F}$ and $\boldsymbol{AXB}+\boldsymbol{CXD}=\boldsymbol{F}$ [25], though it is successful at solving $\boldsymbol{Ax}=\boldsymbol{b}$, $\boldsymbol{AXB}=\boldsymbol{F}$ and the coupled Sylvester matrix equations [21, 22]. How to find a way to overcome these obstacles is the main motivation of this paper.

The symmetric positive definite systems constitute one of the most important cases of special $\boldsymbol{Ax}=\boldsymbol{b}$ problems [29]. To solve this kind of matrix equations, the properties of the coefficient matrices have been investigated and these properties play important roles in the analysis of the convergence of the related iterative algorithms [5]. Closely related to the symmetric positive definite matrix, there is a kind of matrices and their eigenvalues have special structure and bounds which are encountered in the topics of the Jacobi iteration [6] and the preconditioning of the conjugate gradient method [5]. To our best knowledge, the properties of the eigenvalues related to this kind of matrices have not been fully investigated. This paper discusses the structure

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