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Disturbance tolerance and rejection of discrete-time stochastic systems with saturating actuators

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Abstract

This paper considers the problems of disturbance tolerance and disturbance rejection capabilities of discrete-time stochastic systems with saturating actuators. We consider two classes of disturbances whose energy and magnitudes are bounded. A simple condition is given under which any trajectories starting from a certain ellipsoid will remain inside an outer ellipsoid. Invariant sets of criteria are presented. Based on the derived condition, the problems of disturbance tolerance and rejection capabilities of the closed-loop system under a given state feedback law are solved. The effectiveness of these results is demonstrated by a simulation example.

1. Introduction

In recent years, the problems of analysis and synthesis of disturbances for linear systems subject to actuator saturation have been addressed [2]. In [3], the disturbance tolerance and rejection capabilities are studied for a class of linear systems with actuator saturation and $\mathcal{L}_2-$disturbances. The evaluation of the disturbance tolerance and rejection capabilities of the closed-loop system with imprecise knowledge of actuator input output characteristics is given in [1], where a linear matrix inequality (LMI) approach is proposed. The results in [1] were extended to the singular linear system case in [5]. The problem of disturbance attenuation is investigated in [4,7], respectively. It is noted that most of these articles are about continuous systems subject to actuator saturation. However, the study of discrete-time systems is also of both practical and theoretical importance [6,8–14]. The framework of discrete-time plants with

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saturation nonlinearity is considered in [15], where the problem of disturbance rejection with saturating actuators is addressed. A method of estimating $\mathcal{L}_\infty$ norm of an output signal of feedback systems with saturation nonlinearity is developed in [16]. Furthermore, the problem of robust H_∞ guaranteed cost control law design for uncertain Markovian jump systems with distributed delays and input delays is studied in [17]. The authors in [18] consider the problem of parameter-dependent robust stability analysis for uncertain Markovian jump systems with time-varying delay. The problems of robust stabilization and robust H_∞ control for uncertain discrete stochastic systems with time delays are investigated in [19]. However, saturation is not considered. In [1], the authors study the robustness of continuous systems with respect to the uncertainties in the actuator input output characteristics, but stochastic uncertainty is not considered.

In this paper, we are concerned with the problems of disturbance tolerance and rejection capabilities of discrete-time stochastic systems with saturating actuators. Firstly, we present sufficient conditions in terms of LMIs, under which the trajectories of the given system starting from a bounded set remain bounded for any disturbance in $\mathcal{W}_\alpha^2$ or $\mathcal{W}_\alpha^\infty$. For the given system, the disturbance tolerance capability is measured by the maximal energy/magnitude bound. Furthermore, the disturbance rejection capability can be measured by the restricted $\mathcal{L}_2$ gain from the disturbance to the system output or $\mathcal{L}_\infty$ norm of the system output.

The remainder of this paper is organized as follows. Sections 2 and 3 provide some preliminary materials and characteristics of the bounded state stability. In Sections 4 and 5, the disturbance tolerance and rejection capabilities are investigated, respectively. Simulation results are given in Section 6. Section 7 concludes the paper.

Notation: R stands for the set of real numbers. R^n denotes the set of real valued vectors. $R^{m\times n}$ represents the set of real $m \times n$ matrices. The notation F^T represents the transpose of the matrix F. For a matrix $F\in R^{m\times n}$, $\mathcal{L}(F):=\{x(k)\in R^n : |Fx(k)|\le 1\}$. For a symmetric and positive-definite matrix $P\in R^{n\times n}$ and a positive scalar ρ, we let the ellipsoidal set $\varepsilon(P,\rho)=\{x(k)\in R^n : x^T(k) Px(k)\le\rho\}$. Finally, we define $[p,q]=\{p,p+1,\ldots,q\}$ where $p\le q$.

2. Problem formulation

Consider a class of discrete-time stochastic systems with saturating actuators described by

$$\begin{aligned} x(k+1) &= Ax(k)+B\,\mathrm{sat}(u(k))+Ew(k)+Dx(k)v(k),\\ u(k) &= Fx(k),\\ z(k) &= Cx(k), \end{aligned} \tag{1}$$

where $x(k)\in R^n$ is the state vector; $u(k)\in R^m$ is the control input; $w(k)\in R^q$ is the disturbance; $z(k)$ is the output; $v(k)$ is a scalar Wiener process defined on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$. A,B,C,D and E are known constant matrices. It is supposed that

$$\mathbb{E}[v(k)]=0,\quad \mathbb{E}[v^2(k)]=1,\quad \mathbb{E}[v(i)v(j)]=0(i\neq j), \tag{2}$$

where the stochastic process $v(0),v(1),v(2),\ldots$, are assumed to be independent. In the above system, $\mathrm{sat}: R^m\to R^m$ is the saturating function which is defined as $\mathrm{sat}(u(k))=[\mathrm{sat}(u_1(k))\ \mathrm{sat}(u_2(k))\ \cdots\ \mathrm{sat}(u_m(k))]^T$, where one has $\mathrm{sat}(u_i(k))=\mathrm{sign}(u_i(k))\min\{|u_i(k)|,1\}$. We also assume that the disturbance w belongs to one of the following two classes:

$$\mathcal{W}_\alpha^2:=\left\{w: R_+\to R^q : \sum_{k=0}^{+\infty} w^T(k)w(k)\le\alpha\right\}, \tag{3}$$