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# Hybrid modified function projective synchronization of two different dimensional complex nonlinear systems with parameters identification

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**Abstract**

In this article, a novel synchronization scheme is proposed to achieve hybrid modified function projective synchronization (HMFPS) in two different dimensional complex nonlinear systems with fully unknown parameters. In the complex space, the response system are asymptotically synchronized up to the different order's drive system by the state transformation with a scaling function matrix, and all of unknown parameters in both drive and response systems are achieved to be identified. Based on the Lyapunov stability theory, an adaptive controller and updated laws of parameters are developed. Respectively on the ways of increased order and reduced order, the corresponding numerical simulations demonstrate the effectiveness and feasibility of the proposed scheme.


## 1. Introduction

In 1982, the complex Lorenz equations were proposed by Fowler et al. [1], which extended nonlinear systems into complex space. After that, some research works in this field have been achieved [2–8]. With in-depth study of complex nonlinear systems, a variety of physical phenomena could be described by the chaotic or hyperchaotic complex systems, for instance, the detuned laser systems and the amplitudes of electromagnetic fields. In secure communication, the complex nonlinear systems can carry more transmitted information and improve additionally

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security [9–12]. Based on the wide scope of applications, the complex nonlinear systems have been receiving more attentions.

After the seminal work on chaos synchronization proposed by Pecora and Carroll [13], in the past two decades, different synchronization schemes have been proposed such as complete synchronization (CS) [14,15], lag synchronization (LS) [16,17], generalized synchronization (GS) [18], projective synchronization (PS) [19], function projective synchronization (FPS) [20,21], etc. Recently, researchers proposed modified function projective synchronization (MFPS) as a generalization of FPS, in which the response system could be synchronized up to the drive system with a scaling function matrix [22–25]. However, most of the above-mentioned synchronization schemes are based on the same order of the drive and response systems. In reality, especially for chemical and biological science, it is normal for two nonlinear systems with different orders, which cannot stop the synchronization happen. For example, the physiological synchronization phenomenon exists between higher-order and lower-order thalamic neurons as well as between the circulatory and respiratory systems [26]. Therefore, the study of synchronization schemes between different order nonlinear systems is important and necessary from the perspective of both control theory and practical application. Recently in Refs. [27–29], researchers investigated synchronization schemes in two different order's chaotic systems, but discussions were not comprehensive that only increased-order or reduced-order was involved; Wang et al. [30] proposed projective synchronization in two different orders' systems by reduced and increased order, respectively, however, the discussion under the assumption that all the parameters are known. Furthermore, the above works are all achieved in real space.

Due to abundant potential applications in detuned laser, rotating fluids, waves and turbulence in chemical oscillations, electronic circuits and particle beam in high energy accelerators, synchronization in complex nonlinear systems attracts the attention of researchers. In Refs. [31,32], Mahmoud et al. proposed complete synchronization and anti-synchronization between chaotic complex systems. Nian et al. [33] researched the module-phase complete synchronization, and Liu et al. [34] proposed anti-synchronization in two chaotic complex systems with unknown parameters. As a generalization of synchronization, depending on the form of the scaling function matrix, MFPS covers different kinds of synchronization schemes, such as CS, GS, PS, MPS, and FPS. So, it is meaningful and valuable to propose a general method to achieve MFPS in chaotic complex systems. Liu et al. [35] studied the adaptive MFPS in two general unknown chaotic complex systems and provided some reasonable results. However, all of the above studies are only related to the models with the same dimensions in drive and response systems and do not involve the situation of different dimensional systems.

Motivated by the above discussion, we propose a novel synchronization scheme, hybrid modified function projective synchronization (HMFPS) for two different dimensional chaotic complex systems with fully unknown parameters. At the best of our knowledge, up to now, there are no published results about this field. It should be noted that, compared with MFPS, HMFPS is a further generalization. In MFPS, the scaling function matrix is a diagonal matrix and each state variable of drive system is synchronized with a single state variable in response system up to a special scaling function. However, in this article, by means of state transformation, multiple state variables in response system with respective scaling functions will be involved for a corresponding state variable of drive system, which can provide higher complexity for some applications in the fields of secure communication, biological chemistry and neuroscience, such as multichannel digital secure communications, neuronal activity in human brain, the circulatory and respiratory systems, etc. Moreover, to consider the practical situations, the parameters of drive and response systems are fully unknown in our discussion. In this article, the