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# Exponential stability of a class of nonlinear singularly perturbed systems with delayed impulses

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**Abstract**

A class of nonlinear singularly perturbed systems with delayed impulses is considered. By delayed impulses we mean that the impulse maps describing the state's jumping at impulsive moments are dependent on delayed state variables. Assuming that each of two lower order subsystems possesses a Lyapunov function, exponential stability criteria for all small enough values of singular perturbation parameter are obtained. It turns out that the achieved exponential stability is robust with respect to small impulse input delays. A stability bound on perturbation parameter is also derived through using those Lyapunov functions. Additionally, for a class of singularly perturbed Lur'e systems with delayed impulses, an LMI-based method to determine stability and an upper bound of the singular perturbation parameter is presented. The results are illustrated by an example for the position control of a dc-motor with unmodelled dynamics.


## 1. Introduction

Many physical systems contain two significantly different dynamics: fast dynamics and slow dynamics. Such systems can be modeled by singularly perturbed systems involving several small parameters [1,2]. Compared with regular dynamical systems, a distinguish feature of singularly perturbed systems is that when the singular perturbation parameter is taken to be zero, the order of the system is lower than that for other parameter values. From the above reason, a so-called reduction technique is usually used to analysis and control of singularly perturbed systems. In the framework of reduction technique, the original full system is firstly decomposed into two lower-order subsystems, i.e., the boundary layer and the reduced ones. Then one tries to

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determine the behavior of the full-order system from the behavior of these two lower order subsystems.

For stability analysis of singularly perturbed systems, many important results have been reported in the literature (see, e.g., [3–15] and the references therein). On the other hand, impulsive differential systems have found important applications in various fields, such as control systems with communication constraints [16–18], sampled-data systems [19,20], mechanical systems [21,22], etc. A stability analysis of nonlinear singularly perturbed systems with impulse effects was first given in [23]. A similar result can be found in [24]. Unlike the stability analysis using scalar Lyapunov functions for singularly perturbed systems without impulse performed in [3,5], the method proposed in [23] is based on vector Lyapunov functions and reduction technique. The use of vector Lyapunov functions in singularly perturbed impulsive systems can provide a more flexible framework since each component of the vector Lyapunov function needs to satisfy less rigid conditions as compared to scalar Lyapunov functions. Recently, this method has been extended to study robust stability of uncertain singularly perturbed impulsive systems [25] as well as exponential stability of singularly perturbed time delay systems with impulse effect [26,27]. It is worth pointing out that in [23–27], the impulse maps describing the state's jumping at impulsive moments are assumed to be only dependent on the instantaneous states at the same impulsive moments. However, in some engineering applications, the impulse may rely on the delayed state. For instance, in recent years, impulsive control strategy has been widely studied and gradually become an important control approach for nonlinear dynamical systems [36]. From the viewpoint of control theory, the impulses are samples of the state variables of the controlled system at discrete moments. When the sampled impulses are transmitted through a digital communication network, transmission delays are unavoidable due to factors such as communication interference or congestion, computation time, data packet dropout, etc. Thus, the introduction of input delays into impulses may lead to more accurate and reasonable description of the impulsive controlled systems. Besides the impulsive controlled systems mentioned above, delayed impulses appear also in some networked control systems with transmission delays. For example, in [37], the authors presented a new networked control system (NCS) model that incorporates communication constraints, varying transmission intervals and varying delays. The new NCS therein was modeled by a nonlinear impulsive system in which the networked error vector is subject to delayed impulses. Because the presence of impulse input delays can degrade the system performance and can even lead to instability. Therefore, it is of importance to ensure the robustness of system stability towards small impulse input delay, preferably in a quantitative manner. We note that for regular nonlinear systems with delayed impulses, there are only few stability results available [28–30,35]. In [28], a system argumentation approach was introduced to establish stability criterion for delay-free autonomous systems with delayed impulses. In [29,30], Razumikhin-type analysis techniques were used to analyze the effects of delayed impulses on system stability of nonlinear time delay systems. In [35], a delay differential inequality based method was developed for stability analysis of a class of neural networks with delayed impulses. Some sufficient conditions for global exponential stability were derived therein. We note that the stability results of [35] require that the continuous dynamics are exponentially stable. To the best of the authors' knowledge, so far no attempting has been made towards solving the stability problem of nonlinear singularly perturbed systems with delayed impulses. This problem remains challenging, thereby, must be tackled.

The aim of the present paper is to systematically investigate the stability of nonlinear singularly perturbed systems with delayed impulses. As in [23,24], the original full-order system with delayed impulses is decoupled into two lower-order subsystems in different time scales.