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# New criterion for finite-time stability of linear discrete-time systems with time-varying delay ☆

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**Abstract**

This paper is concerned with the problem of finite-time stability analysis of linear discrete-time systems with time-varying delay. The time-varying delay has lower and upper bounds. By choosing a novel Lyapunov–Krasovskii-like functional, a new sufficient condition is derived to guarantee that the state of the system with time-varying delay does not exceed a given threshold during a fixed time interval. Then, the corresponding corollary is developed for the case of constant time delay. Numerical examples are provided to demonstrate the effectiveness and merits of the proposed method.


## 1. Introduction

Numerous existing literatures mainly concern about the stability analysis and controller design for the dynamical systems over infinite time interval. From the practical point of view, our interests are focused on the behavior of the system over a prescribed time interval in some cases. For instance, in the presence of saturation or controlling the trajectory of a space vehicle from an initial point to a final one in a prescribed time interval. That is, the time interval is fixed, the state of the system does not exceed a certain bound during this time interval. It is called finite-time stability (FTS) [1] or short time stability. Some early results on FTS [2,3] lack the operative test

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conditions. The problem of FTS has been revisited using linear matrix inequality technique, which allows to find feasible conditions guaranteeing FTS. Ref. [4] investigated the finite-time control of linear systems subject to parametric uncertainties and disturbances. Ref. [5] studied the problem of finite-time stabilization via dynamic output feedback. Refs. [6] and [7] addressed the finite-time control of discrete-time linear systems. The problem of finite-time stabilization was developed for nonlinear systems in [8] and [9]. However, time delay is not considered in all the above results .

In some practical systems (such as chemical engineering processing, neural network, inferred grinding model, etc.), time delay is inevitable, and the delay is always time-varying. This inherent feature of the system always causes instability and leads to unsatisfactory performance. Therefore, the study for the stability and stabilization of systems with time-varying delay is of significance. Refs. [10–16] investigated the stability criteria for systems with time-varying delays. Refs. [17–20] developed the stabilization conditions for systems with time-varying delay.

However, most of the existing results concentrate on the asymptotical stability, exponential stability or other problems, not the finite-time stability. In [25], San Filippo and Dorato had given a longitudinal flight control example to illustrate the wide practical use of the theoretic results on finite time stability. Therefore, the research on finite time stability is of great practical significance, which is also the motivation of our study. The study of finite-time stability and stabilization for systems with time-delay has received a lot of attention in recent years [21–23], but the time-delay to be considered is constant, not time-varying. On the other hand, it should be emphasized that the definition of the finite-time stability in [26,27] is different from what we investigate. These two papers used the FTS defined in [28], which requires the state trajectory should converge to the equilibrium in a finite time interval. On the other hand, these results mainly focus on the continuous-time systems, while little consideration has been taken on the discrete-time systems with time-varying delay. Ref. [24] investigated the finite-time control for discrete-time systems with time-varying delay.

The main contribution of this paper is that a new finite-time stability criterion of linear discrete-time system with time-varying delay is presented. We select a novel Lyapunov–Krasovskii-like functional, and present a sufficient condition to guarantee that the state of the system does not exceed a certain bound during a prescribed time interval. For the final part of the Lyapunov–Krasovskii-like functional, we can select different values for the positive definite matrices in this paper. However, there are some structural restrictions for the Lyapunov matrix in [24]. Moreover, some free-weighting matrices [29] are introduced to handle the useful items in the derivation process. It is shown that we can obtain better performance than that in [24] by numerical examples.

The rest of the paper is organized as follows. In Section 2, the considered system is stated, and some preliminaries are provided. In Section 3, by selecting a novel Lyapunov–Krasovskii-like functional, a sufficient condition is presented to guarantee the finite-time stability of linear discrete-time systems with time-varying delay, which is the main result of this paper. Section 4 gives numerical examples to show the advantage of the developed results. Finally, in Section 5, some conclusions are drawn.

Notations. $R^n$ and $R^{n\times m}$ denote the $n$-dimensional Euclidean space and the set of $n\times m$ real matrices. $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ are the maximum and the minimum eigenvalues, respectively. $N^+$ represents the set of positive integers. In addition, in symmetric block matrices, we use $*$ as an ellipsis for the term that is induced by symmetry. $diag\{\ldots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P>0$ ($\geq 0$) means $P$ is symmetric and positive definite (positive semi-definite). $I$ and 0 represent identity matrix and zero matrix, respectively.

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