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A collocation method using Hermite polynomials for approximate solution of pantograph equations

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Abstract

In this paper, a numerical method based on polynomial approximation, using Hermite polynomial basis, to obtain the approximate solution of generalized pantograph equations with variable coefficients is presented. The technique we have used is an improved collocation method. Some numerical examples, which consist of initial conditions, are given to illustrate the reality and efficiency of the method. In addition, some numerical examples are presented to show the properties of the given method; the present method has been compared with other methods and the results are discussed.

1. Introduction

Functional–differential equations with proportional delays are usually referred to as pantograph equations or generalized pantograph equations. The name *pantograph* originated from the work of Ockendon and Tayler [1] on the collection of current by the pantograph head of an electric locomotive.

In recent years, there has been a growing interest in the numerical treatment of pantograph equations of the retarded and advanced type [2,3]. A special feature of this type is the existence of compactly supported solutions [4]. This phenomenon was studied in [5] and has direct applications to approximation theory and to wavelets [6].

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Pantograph equations are characterized by the presence of a linear functional argument and play an important role in explaining many different phenomena. In particular they turn out to be fundamental equations when ODEs-based model fails. These equations arise in industrial applications [1,7] and in studies based on biology, economy, control and electrodynamics [8,9].

The Taylor method has been used to find the approximate solutions of differential, difference, integral and integro-differential-difference, multi-pantograph and generalized pantograph equations [10–18]. The basic motivation of this work is to apply the Hermite method to the nonhomogenous and the homogenous generalized pantograph equations and multi-pantograph equations given in [4,18–20].

In this paper, a new Hermite collocation method is developed to find approximate solutions for the generalized pantograph equations with variable coefficients under the mixed conditions. These solutions are obtained in terms of Hermite polynomial. Also in Section 5, examples of these kinds of equations are solved using this new method and the results are discussed.

Our purpose in this study is to develop and to apply a new Hermite collocation method to the pantograph equation

$$y^{(m)}(t) = \sum_{j=0}^{J}\sum_{k=0}^{m-1} P_{jk}(t)y^{(k)}(\lambda_{jk}t) + g(t), \quad 0 \le t \le 1 \tag{1}$$

which is a generalization of the pantograph equations given in [4,19,20] with the mixed conditions

$$\sum_{k=0}^{m-1} a_{ik}y^{(k)}(t) = \lambda_i, \quad i = 0,1,\ldots,m-1. \tag{2}$$

Here a_{ik}, λ_i, λ_{jk} are real or complex coefficients, $P_{jk}(t)$ and $g(t)$ are continuous functions defined in the interval $0 \le t \le 1$.

We assume that the solution is expressed in the truncated Hermite series form

$$y(t) = \sum_{n=0}^{N} a_n H_n(t), \quad H_n(t) = (-1)^n e^{t^2} \frac{d^n}{dt^n}(e^{-t^2}) \tag{3}$$

so that the Hermite coefficients to be determined are a_n, $n = 0,1,2,\ldots,N$, $N \in IN$.

In the view of the derivation relation of Hermite polynomials

$$H_n'(t) = 2nH_{n-1}(t),$$

we obtain the following matrix relation between the matrices $H^{(k)}(t)$ and $H(t)$:

$$\underbrace{\begin{bmatrix} H_0'(t) \\ H_1'(t) \\ H_2'(t) \\ H_3'(t) \\ \vdots \\ H_{N-1}'(t) \\ H_N'(t) \end{bmatrix}}_{[\mathbf{H}'(t)]^T} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 2\cdot 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 2\cdot 2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 2\cdot 3 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2(N-1) & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 2N & 0 \end{bmatrix}}_{\mathbf{M}} \underbrace{\begin{bmatrix} H_0(t) \\ H_1(t) \\ H_2(t) \\ H_3(t) \\ \vdots \\ H_{N-1}(t) \\ H_N(t) \end{bmatrix}}_{[\mathbf{H}(t)]^T}$$