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Improved delay-dependent exponential stability criteria for time-delay system

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Abstract

This paper considers the problem of time delay-dependent exponential stability criteria for the time-delay linear system. Utilizing the linear inequality matrices (LMIs) and slack matrices, a novel criterion based on the Lyapunov–Krasovskii methodology is derived for the exponential stability of the time-delay system. Based on the criteria condition we concluded that the upper bound of the exponential decay rate for the time-delay system can be easily calculated. In addition, an improved sufficient condition for the robust exponential stability of uncertain time-delay system is also proposed. Numerical examples are provided to show the effectiveness of our results. Comparisons between the results derived by our criteria and the one given in Liu (2004) [1], Mondie and Kharitonov (2005) [2], and Xu et al. (2006) [3] show that our methods are less conservative in general. Furthermore, numerical results also show that our criteria can guarantee larger exponential decay rates than the ones derived by Liu (2004) [1] and Mondie and Kharitonov (2005) [2] in all time delay points we have tested and in some of time delay points obtained by Xu et al. (2006) [3].

1. Introduction

Time delay phenomenons existed in most of the control systems and may cause unexpected performance, such as the instability of the system. In such a case, analyzing the performance and stability conditions of time delay system plays an important role in system design and control. Over the past many decades, the stability conditions of time delay systems have been thoroughly researched [1–16,19–23]. Among several different

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techniques, the Lyapunov second method [17], which makes use of the Lyapunov function and its first-order derivative instead of requiring the knowledge of the system general solution, has been comprehensively studied. In recent several decades, the stability criteria based on linear inequality matrices (LMIs) became popular due to that when formulated in terms of LMI, the control problem can be efficiently solved with convex optimization algorithms comparing with conventional techniques. The stability of linear time delay system, which is the fundamental but important in system control, has attracted much interest over a half century. Representative results of stability conditions for linear time delay control system based on linear inequality matrices can be found in [1–6] and references therein. Since most of the research papers concerned the asymptotical stability conditions of the linear systems in the past decades, exponential stability conditions of linear systems which guarantees stronger stability constraints than the asymptotical stability are urgent. Some of the pioneer work on the analysis of exponential stability for linear systems can be found in [1–3]. In [1], the author first developed the asymptotical stability condition and then extended to exponential case by introducing the Schur complements [18] into the Lyapunov–Krasovskii function. A modification of standard LMI-type stability which allows to obtain the exponential decay rate of the system solution was shown in [2]. By choosing an appropriate Lyapunov–Krasovskii function, an improved exponential stability condition for time-delay systems has been proposed in [3]. As time delays existed in control systems may lead to the instability of the system, a large number of recent papers have been reported on the time delay dependent stability conditions [1,6,8,10,11,14–16]. Generally, a large time delay condition that can stabilize the system would be less conservative than the one with a small time delay. Recent research has shown that introducing modulatory matrices into the derivative of Lyapunov–Krasovskii function would be helpful to reduce some conservatism in the existing delay-dependent stability conditions and the derivative would remain unaffected. Hence, the flexibility of choosing the slack matrix would result in a less conservative stability condition and a relatively large time delay.

In this paper, we study the problem of time delay-dependent exponential stability criteria for the time-delay linear system. Using the recent popular linear inequality matrices (LMIs) and utilizing the Newton–Leibniz formula and slack matrices, a novel criterion based on the Lyapunov–Krasovskii methodology is derived for the exponential stability of the time-delay system. Based on the criteria condition we concluded that the upper bound of the exponential decay rate for the time-delay system can be easily calculated. In addition, an improved sufficient condition for the robust exponential stability of uncertain time-delay system is also proposed. Numerical examples are provided to show the effectiveness of our results. Comparisons between the results derived by our criteria and the one given in [1–3] show that our methods are less conservative in general. Furthermore, numerical results also show that our criteria can guarantee larger exponential decay rates than the ones derived by [1,2] in all the delay points we have tested and in some of the delay points obtained by [3].

Notation: For expression simplicity, throughout this paper, $Q>0$ and $Q<0$ mean that the matrix Q is a positive definite matrix and a negative definite matrix, respectively. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum eigenvalue and the maximum eigenvalue of the matrix A, respectively. $\|\varphi\|_h = \sup_{\theta\in[-h,0]}\{\|\varphi(\theta)\|\}$, $\|x(t)\| = (\sum_{i=1}^{n} x_i^2(t))^{1/2}$, and $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$.