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New results on delay-dependent stability analysis for neutral stochastic delay systems

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Abstract

This paper is concerned with the problem of stability analysis for neutral stochastic delay systems. Firstly, expectations of stochastic cross terms containing the Itô integral are investigated by the martingale theory. Based on this, an improved delay-dependent stability criterion is derived for neutral stochastic delay systems. In the derivation process, the mathematical development avoids bounding stochastic cross terms, and neither the model transformation method nor free-weighting-matrix method is used. Thus the method leads to a simple criterion and shows less conservatism. Finally, two examples are provided to demonstrate the effectiveness and reduced conservatism of the proposed conditions.

1. Introduction

Time delays arise in various practical systems and they are often a source of instability and poor performance. Thus, problems of stability and control of time-delay systems have been of great importance and interest. Recently, much attention has been focused on delay dependent conditions for the analysis and control of time-delay systems, because delay-dependent

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conditions are generally less conservative than delay-independent ones. Many effective methods have been proposed such as the discretized Lyapunov–Krasovskii functional method [1], the model transformation method [2–5], the free-weighting-matrix method [6–9], the Finsler projection approach [10], and so on.

On the other hand, some practical systems such as chemical engineering systems, distributed networks containing lossless transmission lines, can be modeled by using the model of neutral-type delay systems [35]. Due to the fact that the neutral delay system involves the delays in both its state and the derivative of the state, stability of neutral delay systems proves to be a more complex issue, and delay-dependent stability problems of neutral delay systems have received considerable attention by using above methods during the past decades [1,3,5,6,8,11–14]. Recently, since systems in the real world are always perturbed by stochastic noises, Kolmanovskii and Nosov [36,37] introduced the neutral stochastic differential functional equations to represent mathematical models for practical systems such as chemical engineering systems and the theory of aeroelasticity. And neutral stochastic delay systems have been studied over recent years [15–19]. For instance, the delay-dependent stability problems were addressed in [18,19] by the free-weighting-matrix method, while the $\mathcal{H}_\infty$ control problem was investigated in [16]. Moreover, for nonlinear stochastic neutral systems, delay-dependent stability results were obtained in [17]. Although these results given are effective, there are still some problems to be pointed out for neutral stochastic delay systems, even these problems exist in stochastic delay systems.

- The Newton–Leibniz formula is still valid in stochastic case?

Recently, some papers such as [20] used the Newton–Leibniz formula to obtain the delay-dependent stability condition for stochastic delay systems. Unfortunately, from the preface of [25], we can know that the Newton–Leibniz formula is not valid in stochastic differential equations. In fact, for the following neutral stochastic functional differential equation:

$$d[x(t)-D(x_t)] = f(t,x_t)\,dt + g(t,x_t)\,dw(t) \tag{1}$$

on $t \geq 0$ with the initial data $x_0 = \{x(\theta) : -h \leq \theta \leq 0\} = \xi \in C^b_{\mathcal{F}_0}([-h,0];\mathcal{R}^n)$, it is known in [15,25] that Eq. (1) is a symbolic differential form and is interpreted as meaning the stochastic integral equation

$$x(t)-D(x_t) = x(0)-D(x_0) + \int_0^t f(s,x_s)\,ds + \int_0^t g(s,x_s)\,dw(s). \tag{2}$$

- How to deal with stochastic cross terms containing the Itô integral?

Since the Newton–Leibniz formula is not valid in stochastic case, both of the model transformation method and the free-weighting-matrix method must use the stochastic integral equation (2) to obtain delay-dependent conditions. Then there will be unavoidable to appear the following stochastic cross-terms containing the Itô integral: $x(t)^T M \int_{t-h}^t g(s,x_s)\,dw(s)$, $x(t-h)^T N \int_{t-h}^t g(s,x_s)\,dw(s)$, $(\int_{t-h}^t f(s,x_s)\,ds)^T L \int_{t-h}^t g(s,x_s)\,dw(s)$. It is still very difficult to calculate the expectations of these stochastic cross terms. The existing results in [18,21–23] resorted to bounding techniques to deal with stochastic cross terms, which obviously can bring unavoidable conservatism. For these stochastic cross