

Short communication

A linear algebraic criterion for controllability of both continuous-time and discrete-time symmetric bilinear systems[☆]

Lin Tie*, Kai-Yuan Cai, Yan Lin

School of Automation Science and Electrical Engineering, Beihang University (Beijing University of Aeronautics and Astronautics), 100191 Beijing, PR China

Received 26 June 2012; received in revised form 23 November 2012; accepted 11 January 2013

Available online 21 January 2013

Abstract

This paper presents a necessary and sufficient condition for controllability of two-dimensional continuous-time and discrete-time symmetric bilinear systems. The presented condition is a linear algebraic version which is easy to apply and verify. In particular, for the continuous-time case, the necessary and sufficient condition can replace the classical Lie-algebraic one. For the discrete-time case, the necessary and sufficient condition is new.

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1. Introduction

In this paper, we consider the unconstrained symmetric bilinear system [1]

$$\dot{x}(t) = (u_1(t)B_1 + \cdots + u_m(t)B_m)x(t) = \sum_{i=1}^m u_i(t)B_i x(t), \quad (1)$$

where $x(t) \in \mathbb{R}^n$, $u_1(t), \dots, u_m(t) \in \mathbb{R}$ are piecewise constant and $B_1, \dots, B_m \in \mathbb{R}^{n \times n}$, and its discrete-time counterpart by Euler discretization [1,2]

$$x(k+1) = (I + u_1(k)B_1 + \cdots + u_m(k)B_m)x(k) = \left(I + \sum_{i=1}^m u_i(k)B_i \right) x(k), \quad (2)$$

[☆]This work was supported by the China Postdoctoral Science Foundation funded project under Grants 2011M500216, 2012T50035, the National Nature Science Foundation of China under Grants 61203231, 61273141, and the Program for Excellent Doctoral Dissertation Supervisor of Beijing under Grant 20121000602.

*Corresponding author.

E-mail address: tielinllc@gmail.com (L. Tie).

where $x(k) \in \mathbb{R}^n$ and $u_1(k), \dots, u_m(k) \in \mathbb{R}$. Without loss of generality, $B_1, \dots, B_m$ are always assumed to be linearly independent. Note that system (1) can be derived from autonomous linear systems if considering the parameters as control inputs. System (1) can also be seen from switched linear systems if $u_1(t), \dots, u_m(t)$ satisfy a switching law.

Indeed, bilinear systems are good models to represent many real-world processes ranging from engineering to non-engineering fields [1–5]. Besides, they can be used to approximate complex nonlinear systems that the classical linear systems do not fit [6]. Over the past decades, there has been a great interest in the theory and applications of bilinear systems [1]. Controllability of continuous-time bilinear systems has thus received considerable attention, among which the unconstrained symmetric bilinear systems are thought of to have the most complete theory. For controllability of discrete-time bilinear systems, however, only a few works have been done. Here, system (1) (system (2)) is said to be controllable if for any ξ, η in $\mathbb{R}_*^n$ ($\mathbb{R}_*^n := \mathbb{R}^n \setminus \{0\}$), there exist control inputs such that ξ can be transferred to η .

There already exists a necessary and sufficient Lie-algebraic criterion on controllability of system (1),¹ where the theory of Lie groups and Lie algebras has played an important role. The necessary and sufficient Lie-algebraic criterion says that system (1) ($n \geq 2$) is controllable if and only if the Lie algebra $\{B_1, \dots, B_m\}_L$ generated by $B_1, \dots, B_m$ is transitive [1]. However, controllability of system (2) is unknown. In this paper, we focus on the systems in dimension two. We first derive a linear algebraic criterion on controllability of system (1) instead of the Lie-algebraic one, which is easy to apply and verify. We then show that the derived linear algebraic criterion also works for system (2). As a result, system (1) will be controllable if and only if its discrete-time counterpart is controllable. Finally, we make discussions on controllability of the systems with high dimensions.

2. Continuous-time case

We introduce a necessary condition on controllability, which is valid for both systems (1) and (2).

Theorem 1. Consider the system (1) (system (2)) with $m > 1$, $n > 1$. If the system is controllable, then $B_1, \dots, B_m$ have no common real eigenvector.

Proof. If $B_1, \dots, B_m$ have a common real eigenvector, named ξ , then it can be seen that the linear subspace $\text{span}\{\xi\}$ is invariant for the system (1) (system (2)). That is, any state initiated from $\text{span}\{\xi\}$ will not leave it, which makes the system (1) (system (2)) uncontrollable. $\square$

For sufficient conditions for system (1) or (2) to be controllable, the necessary one of Theorem 1 will suffice if the system dimension is two. We first prove this for system (1) by using the following result.

Theorem 2 (Koditschek and Narendra [7]). For any linearly independent $B_1, B_2 \in \mathbb{R}^{2 \times 2}$, the Lie algebra $\{B_1, B_2\}_L$ generated by B_1, B_2 is transitive if and only if B_1, B_2 have no common real eigenvector.

¹For the study on controllability of systems (1) and (2), one can go to Appendix A.

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