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Bifurcation analysis in the control of chaos by extended delay feedback ☆

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Abstract

Bifurcation scenario in control of chaos with extended delay feedback is considered. Lorenz system is investigated as a neutral delay differential equation. Stability and Hopf bifurcation properties are obtained which show that chaos vanishes at a backward Hopf bifurcation and again appears after a forward Hopf bifurcation with the increasing of time delay, and that chaos is hardly detected under feedback with large delay. We find the branch of Hopf bifurcation being interrupted by chaotic attractor. Extended delay feedback is compared with the traditional delay feedback. These results are obtained in either theoretical or numerical way which provides a clear interpretation for control of chaos with extended delay feedback. Finally some simulations are carried out and applications are given with respect to Makey–Glass equation and Rössler system.

1. Introduction

Chaotic attractor arises a lot in nonlinear dynamical systems since it was first studied by Lorenz [1]. Thereafter more kinds of chaos are observed such as Rössler attractor or Chua's attractor [2,3]. Control of chaos, generally speaking, bringing order to chaotic system, is an interesting and challenging subject in the nonlinear science. So far there are mainly two kinds of control established, namely, the nonfeedback control [4,5] and the

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feedback control proposed by [6–8]. In [9,10], extended delay feedback method, a modification of the traditional delay feedback control technique that allows one to stabilize unstable periodic orbits over a large domain of parameters, was established as the method of extended time delay autosynchronization, which is actually a continuous feedback loop incorporating information from a sequence of previous states. When feedback controller eliminates chaos, there must be some bifurcations occurring in the system, simultaneously. However, so far as we know, bifurcation analysis in a chaotic system with extended delay feedback has not been well studied.

In this paper, we mainly analyze the bifurcation behavior in the control of chaos with extended delay feedback. For the sake of clear expressions of our results, we study the extended feedback in the classical Lorenz equations as

$$\begin{aligned} \dot{x}_1 &= -\beta x_1 + x_2 x_3 \\ \dot{x}_2 &= -\rho x_2 + \rho x_3 \\ \dot{x}_3 &= -x_1 x_2 + \sigma x_2 - x_3. \end{aligned} \tag{1}$$

This Lorenz system is a simplified mathematical model from atmospheric convection, where three variables x_1, x_2 and x_3 are dimensionless. To investigate the extended delay feedback qualitatively, in this paper we assume x_2 to be an observable, because it brings no more nonlinear terms thus does not raise the difficulty of calculations, that we add feedback into the second equation. Lorenz equation with extended delay feedback has the form

$$\begin{aligned} \dot{x}_1 &= -\beta x_1 + x_2 x_3 \\ \dot{x}_2 &= -\rho x_2 + \rho x_3 + k[S(t-\tau) - x_2] \\ \dot{x}_3 &= -x_1 x_2 + \sigma x_2 - x_3, \end{aligned} \tag{2}$$

where the extended feedback $S(t)$ satisfies

$$S(t) = (1-p)x_2 + pS(t-\tau), \tag{3}$$

with $p \in (0,1)$ and $k > 0$. After a sequence of iterations, Eq. (2) together with feedback (3) is an infinite delay differential equation, which is equivalent to

$$\begin{aligned} \dot{x}_1 &= -\beta x_1 + x_2 x_3 \\ \dot{x}_2 - p\dot{x}_2(t-\tau) &= -(\rho + k)x_2 + \rho p x_2(t-\tau) + \rho x_3 - \rho p x_3(t-\tau) + k x_2(t-\tau) \\ \dot{x}_3 &= -x_1 x_2 + \sigma x_2 - x_3. \end{aligned} \tag{4}$$

This is a neutral delay differential equation and can be analyzed by the newly developed method given by [11–14]. More precisely, the main method is from the viewpoint of bifurcation analysis and normal form method. Normal form method of neutral equation is developed since Weedermann [12,13] and application to Hopf bifurcation is studied by [14]. This method is motivated by the formal adjoint theory given in Hale [15], the center manifold theory in [16–18] and the algorithm such as in [19–22]. With the help of bifurcation analysis we give a clear transition from chaos to order as a backward Hopf bifurcation. We also notice that Hopf bifurcation branches are divided into two parts by chaotic attractors. Simultaneously, chaos switches are observed with the appearance of stability switches.

This paper is organized as follows. Section 2 is a standard analysis of the stability and Hopf bifurcation where sufficient conditions to suppress unstable equilibria are given. In Section 3 we obtain the properties of Hopf bifurcation in this neutral equation by using