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Adaptive reduced-basis generation for reduced-order modeling for the solution of stochastic nondestructive evaluation problems

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Abstract

A novel algorithm for creating a computationally efficient approximation of a system response that is defined by a boundary value problem is presented. More specifically, the approach presented is focused on substantially reducing the computational expense required to approximate the solution of a stochastic partial differential equation, particularly for the purpose of estimating the solution to an associated nondestructive evaluation problem with significant system uncertainty. In order to achieve this computational efficiency, the approach combines reduced-basis reduced-order modeling with a sparse grid collocation surrogate modeling technique to estimate the response of the system of interest with respect to any designated unknown parameters, provided the distributions are known. The reduced-order modeling component includes a novel algorithm for adaptive generation of a data ensemble based on a nested grid technique, to then create the reduced-order basis. The capabilities and potential applicability of the approach presented are displayed through two simulated case studies regarding inverse characterization of material properties for two different physical systems involving some amount of significant uncertainty. The first case study considered characterization of an unknown localized reduction in stiffness of a structure from simulated frequency response function based nondestructive testing. Then, the second case study considered characterization of an unknown temperature-dependent thermal conductivity of a solid from simulated thermal testing. Overall, the surrogate modeling approach was shown through both simulated examples to provide accurate solution estimates to inverse problems for systems represented by stochastic partial differential equations with a fraction of the typical computational cost.

Keywords: Material characterization; Nondestructive evaluation; Computational inverse mechanics; Uncertainty quantification; Reduced order model; Sparse grid collocation

1. Introduction

There is a large number of important inverse problems in engineering mechanics, covering applications from material characterization to design of complex physical systems, and a corresponding large amount of work involving

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the solution of these problems. Of particular interest to the present work are inverse problems relating to the use of nondestructive evaluation (NDE) to evaluate various properties of different in-service structures/systems [1–4]. One effective approach for solving such inverse problems in mechanics has been to use a numerical analysis tool, such as finite element analysis, to predict the forward response of the system under consideration and then combine nonlinear optimization to find the unknown properties to best match the response of the numerical model with the desired or measured response of the system [5–10]. Although various research efforts have been directed towards such computational methods for the solution of inverse problems and have made significant strides, there are still several common challenges, most often relating to the ill-posedness of the inverse problems in the form of nonexistent or non-unique solutions along with the excessive computational expense associated with many solution algorithms. In particular, regardless of the solution approach (e.g., gradient based [11,12] or non-gradient-based optimization [13–15], etc.), solving a NDE problem using a computational solution procedure commonly requires a relatively large number of evaluations of the forward response of the system. Moreover, the computational expense drastically increases if considering uncertainty within the system, since the forward problem then involves a stochastic partial differential equation (SPDE) (which is considerably expensive on its own).

There are several different approaches that have been developed for the solution of SPDEs [16–18]. Whether the approach to address the uncertainty is intrusive (i.e., modifies the deterministic boundary value problem) or non-intrusive (i.e., only uses results from the deterministic boundary value problem), these solution approaches typically require a substantial amount of computational expense. As such, there has been considerable effort to attempt to reduce the computational expense of SPDE solutions, both for intrusive [19–22] and non-intrusive approaches [23,24]. Sparse grid approximation approaches are one particular computationally efficient solution technique that builds a low-cost approximation (i.e., surrogate model) of the SPDE and has shown considerable promise for being used in approximating SPDE solutions [25,18]. The sparse grid methods are non-intrusive, and therefore, easy to implement, requiring only the solution of uncoupled deterministic problems, and use substantially fewer evaluations of the boundary value problem in comparison to the traditional Monte Carlo non-intrusive methods, without sacrificing accuracy [17].

There have also been a variety of approaches developed in recent years to solve inverse problem involving SPDEs (i.e., stochastic inverse problems). For example, [26] presented an approach to solve a stochastic inverse heat conduction problem using the spectral stochastic finite element method as the forward solver (i.e., to solve the SPDEs) within an optimization routine. The work in [27] used the sparse grid collocation method based on the Smolyak algorithm with adaptive refinement based on the importance of the stochastic dimensions to solve stochastic natural convection problems. This work was extended for solving a design problem by using a sparse grid representation of the design variables to convert the stochastic optimization problem into a deterministic optimization problem, and gradient-based optimization was used to solve the design problem with stochastic sensitivity computation [28]. In order to reduce the computational cost for Bayesian solutions of inverse problems, Marzouk et al. [29] introduced a method that combines Karhunen–Love (K–L) representation of the unknown field with spectral methods, in which the K–L representation of a scaled Gaussian process prior defines the uncertainty that is propagated through the forward model with a stochastic Galerkin scheme. More recently, Marzouk et al. [30] presented an efficient numerical method, which used generalized polynomial chaos (gPC) to construct a polynomial approximation of a forward solution and the support of the prior distribution to define a surrogate posterior probability density that can be evaluated at low computational cost.

Reduced-basis-type model reduction approaches that identify the relatively low-dimensional basis that is optimal in some sense for representing the physics of the system of interest have been used to produce efficient and accurate numerical representations for several different applications in mechanics [31–35]. By not replacing the boundary value problem governing the mechanics of interest as would be done with surrogate modeling approaches, reduced-basis ROM techniques are more computationally expensive than surrogate modeling approaches, but are typically capable of more accurate approximations, particularly for extrapolating throughout the space of potential system inputs. This ROM approach has also been recently extended to stochastic problems with the work by Boyaval et al. [19] that created reduced-basis ROMs to estimate the solution of an SPDE. There are different strategies to determine the low-dimensional basis, but the focus of the work herein is on methods that derive the "optimal" basis from a given set of potential fields for the system of interest. These given fields can be either experimentally measured or numerically simulated with different values of the system input parameters, depending on capabilities. There are also different approaches to process these given fields to produce a basis. For example, proper orthogonal decomposition (POD)