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A computational framework for large strain nearly and truly incompressible electromechanics based on convex multi-variable strain energies

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Highlights

- A generalisation of the concept of multi-variable convexity to energy functionals additively decomposed into isochoric and volumetric components is carried out in this work.
- Convexification or regularisation strategies are applied to a priori non-convex multi-variable isochoric functionals, yielding unconditionally stable convex multi-variable isochoric functionals.
- A novel extended Hu–Washizu mixed variational principle for nearly and truly incompressible scenarios is presented in this work. A static condensation procedure is applied in order to condense out element-wise the extra fields.
- The computational framework for the three-field mixed variational principle in nearly and truly incompressible scenarios is also presented.
- A Petrov–Galerkin stabilisation technique is applied on the three-field formulation for the circumvention of the LBB condition, enabling the use of linear tetrahedral finite elements for the interpolation of the unknowns of the problem.

Abstract

The series of papers published by Gil and Ortigosa (Gil and Ortigosa, 2016; Ortigosa and Gil, 2016, 0000) introduced a new convex multi-variable variational and computational framework for the numerical simulation of Electro Active Polymers (EAPs) in scenarios characterised by extreme deformations and/or extreme electric fields. Building upon this body of work, five key novelties are incorporated in this paper. First, a generalisation of the concept of multi-variable convexity to energy functionals additively decomposed into isochoric and volumetric components. This decomposition is typical of nearly and truly incompressible materials, group which represents the majority of the most relevant EAPs. Second, convexification or regularisation strategies are applied to a priori non-convex multi-variable isochoric functionals to yield physically meaningful convex multi-variable functionals. Third, based on the mixed variational principles introduced in Gil and Ortigosa (2016) in the context of compressible electro-elasticity, a novel extended Hu–Washizu mixed variational principle for nearly and truly incompressible scenarios is presented. From the computational standpoint, a static condensation procedure is applied in order to condense out the element-wise extra fields, the resulting formulation having a comparable cost to the more standard three-field displacement-potential-pressure mixed formulation. Fourth, the computational framework for the three-field mixed variational principle in nearly and truly incompressible scenarios is also presented. In this case, the novelty resides in the consideration of convex multi-variable energy functionals. Ultimately, this

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leads to the definition of new tangent operators for the Helmholtz's energy functional in the specific context of incompressible electro-elasticity. Fifth, a Petrov–Galerkin stabilisation technique is applied on the three-field formulation for the circumvention of the Ladyženskaja–Babuška–Brezzi (LBB) condition, enabling the use of linear tetrahedral finite elements for the interpolation of the unknowns of the problem. Finally, a series of challenging numerical examples is presented in order to provide an exhaustive comparison of the different variational formulations presented in this paper.

Keywords: Dielectric elastomers; Nonlinear electro-elasticity; Material stability; Mixed variational principles; Polyconvexity; Incompressibility

1. Introduction

Dielectric elastomers (DEs) belong to a general class of Electro Active Polymers (EAPs) with remarkable actuation properties [1–4]. Recently, Li et al. [5] have reported an outstanding voltage induced area expansion of 1980% on a Dielectric Elastomer membrane film. In this specific case, the electromechanical instability is harnessed as a means for obtaining these electrically induced massive deformations with potential applications in soft robots, adaptive optics, balloon catheters and Braille displays [5], among others. Moreover, these materials have been successfully applied as generators to harvest energy from renewable sources, such as human movements and ocean waves [6].

Several authors [7–21] have contributed to the development of a well established variational framework for the numerical simulation of electro active materials. Crucial to this variational framework is the definition of well posed constitutive equations, as these materials are prone to develop a variety of electromechanical instabilities [22]. Among these, it is vital to identify those material instabilities associated to the onset of macroscopic instabilities [23–25]. The latter are related to the loss of ellipticity [26,27] and the loss of positive definiteness of the generalised acoustic tensor [23,24,28] and, ultimately, lead to the ill-posedness of the governing equations. From the numerical standpoint, these detrimental features yield [22–24] a pathological mesh dependence behaviour similar to that observed in the modelling of strain localisation [29].

In nonlinear elasticity, ellipticity of the constitutive model is satisfied ab initio by polyconvex [26,27,30–36,36–48] energy functionals. Gil and Ortigosa [49] extended the concept of polyconvexity to finite strain electromechanics, where the more appropriate term multi-variable convexity was adopted. The authors defined a new electro-kinematic variable set including the deformation gradient tensor $\boldsymbol{F}$, its adjoint or co-factor $\boldsymbol{H}$, its determinant J, the Lagrangian electric displacement field $\boldsymbol{D}_0$ and an additional spatial or Eulerian electromechanical variable denoted as $\boldsymbol{d}$. Convexity of the internal energy functional with respect to the elements of this extended set permits an extension of the concept of ellipticity [27] not only to the entire range of deformations but to any applied electric field.

In the present manuscript, the concept of multi-variable convexity is generalised to energy functionals which are additively decomposed into isochoric and volumetric components. Notice that many authors [43,50,51] advocate for this approach in order to model the behaviour of nearly and truly incompressible materials, as is the case of the popular acrylic elastomer VHB 4910. Based on the work in Ref. [49], some strategies to create appropriate convex multi-variable isochoric invariants by incorporating minor modifications to a priori non-convex multi-variable isochoric invariants are presented in this work.

From the numerical standpoint, in the context of incompressible elasticity, some authors resort to interpolation spaces for both geometry and pressure fields which a priori do not satisfy the Ladyženskaja–Babuška–Brezzi (LBB) [52–54] condition but then recover the ellipticity of the problem via appropriate stabilisation techniques [38,44,51,55–61]. In this regard, the Stream-Upwind-Petrov–Galerkin (SUPG) method can be utilised as a robust technique for the circumvention of the LBB condition [62]. On the contrary, other authors resort to Finite Elements discretisations which satisfy the LBB condition, as for instance, the $P2P1$ and $Q2Q1$ *Taylor–Hood* elements [53].

In the context of incompressible electro-elasticity, some authors [63] resort to the well established *B-bar* [64] element for the discretisation of the standard three-field displacement-potential-pressure variational principle. In this element, a bilinear/trilinear (for 2D/3D applications) interpolation of the geometry and electric potential and a constant element by element interpolation of the pressure field is considered. Unfortunately, this choice of interpolation spaces does not satisfy the LBB condition and can potentially lead to spurious pressure modes [65]. The use of pressure smoothing at a postprocessing stage [66] can help alleviate this shortcoming. In conformity with this non-LBB