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Permanence and periodic solutions for an impulsive reaction-diffusion food-chain system with holling type III functional response☆

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Abstract

An impulsive reaction-diffusion periodic food-chain system with Holling type III functional response is presented and studied in this paper. Sufficient conditions for the ultimate boundedness and permanence of the food-chain system are established based on the upper and lower solution method and comparison theory of differential equation. By constructing appropriate auxiliary function, the conditions for the existence of a unique globally stable positive periodic solution are also obtained. Some numerical examples are shown to illustrate our results. A discussion is given in the end of the paper.

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1. Introduction

Reaction-diffusion equations can be used to model the spatiotemporal distribution and abundance of organisms. A typical form of reaction-diffusion population model is

$$\frac{\partial u}{\partial t} = \mathcal{D}\Delta u + uf(x,u),$$

where $u(x,t)$ is the population density at a space point x and time t, $\mathcal{D}>0$ is the diffusion constant, Δu is the Laplacian of u with respect to the variable x, and $f(x,u)$ is the growth rate per capita, which is affected by the heterogeneous environment. Such an ecological model was first considered by Skellam [1], similar reaction-diffusion biological models were also studied by Fisher [2] and Kolmogoroff et al. [3] earlier. In the past two decades, the reaction-diffusion models, especially in population dynamics, have been studied extensively. For example, Ainseba and Aniţa [4] considered a 2×2 system of semilinear partial differential equations of parabolic-type to describe the interactions between a prey population and a predator population. Meanwhile, they obtained some necessary and sufficient conditions for the stabilizability. Recently, Xu and Ma [5] studied a reaction-diffusion predator–prey system with nonlocal delay and Neumann boundary conditions. They have established some sufficient conditions on the global stability of the positive steady state and the semi-trivial steady state. Liu and Huang [6] investigated a diffusive predator–prey model with Holling type III functional response. They got some sufficient conditions for the ultimate boundedness of solutions and permanence of the system, they also studied the existence of a unique globally stable periodic solution. More researches on the reaction-diffusion dynamical systems, please see [7–14].

There are many examples of evolutionary systems which at certain instants are subjected to rapid changes. In the simulations of such processes it is frequently convenient and valid to neglect the durations of rapid changes. The perturbations are often treated continuously. In fact, the ecological systems are often affected by environmental changes and other human activities. These perturbations bring sudden changes to the system. Systems with such sudden perturbations referring to impulsive differential equations, which have attracted the interest of many researchers in the past twenty years since they provided a natural description of several real processes. Process of this type is often investigated in various fields of science and technology, physics, population dynamics [15–17], epidemics [18], ecology, biology, neural networks [19,20], optimal control [21], and so on.

Recently, some impulsive reaction-diffusion models have been investigated [22–24]. Especially, Akhmet et al. [23] considered an impulsive ratio-dependent predator–prey system with diffusion; meanwhile, they obtained some conditions for the permanence of the predator–prey system and for the existence of a unique globally stable periodic solution. Wang et al. [24] generalized the above impulsive ratio-dependent system to an $n+1$ species and got some analogous results. However, food-chain systems exist extensively in the population dynamics; but, the reaction-diffusion food-chain systems have rarely been studied by scholars. Hence, the study of the dynamics on the impulsive reaction-diffusion food-chain system is the aim of this paper.

Motivated by the above works, we present and study the following impulsive reaction-diffusion food-chain system with Holling type III functional response in this paper:

$$\frac{\partial u_1}{\partial t} = \mathcal{D}_1\Delta u_1 + u_1[a_1(t,x) - b_1(t,x)u_1] - \frac{c_1(t,x)u_1^2u_2}{u_1^2 + r_1(t,x)u_2^2}, \tag{1}$$