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Velocity-based approach in non-linear dynamics of three-dimensional beams with enforced kinematic compatibility

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Highlights

- We present velocity-based approach for dynamic analysis of three-dimensional beams.
- Spatial and temporal discretization are based on additive quantities.
- Rotational degrees of freedom are handled using quaternion algebra.
- A special care is taken in deriving discrete kinematic compatibility equations.

Abstract

In the paper we present a new finite-element formulation for the dynamic analysis of geometrically exact three-dimensional beams. We limit our studies to implicit time-integration schemes and possible approaches for increasing their robustness and numerical stability. In contrast to standard displacement-rotation based approach we present here a spatial and temporal discretization based on velocities and angular velocities. To describe the rotational degrees of freedom quaternions are used. The time-integration scheme and the governing equations of the three-dimensional beam are modified accordingly. In the numerical implementation the Galerkin-type discretization is employed to obtain the finite-element formulation of the problem. The result of our studies is simple, but accurate, efficient and robust numerical model.

Keywords: Structural dynamics; Non-linear beams; Kinematic compatibility; Rotational quaternions; Implicit time integration

1. Introduction

The beam models have attached a great attention during decades due to their wide applicability in various engineering fields and new rapidly developing areas, such as biomechanics. Besides the benefits of such models the mathematical theory behind is demanding enough to make this area of research still challenging for researchers, which is reflected in many recent publications (e.g., [1–8]). The problems reported and analysed in literature often

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stem from choosing the spatial rotations as members of the variables describing the configuration space of the beam. Not only that the spatial rotations belong to a non-linear manifold, their behaviour in space and time is non-trivial, which requires a special treatment in numerical formulations.

Despite the well known and widely documented difficulties with finite spatial rotations they are usually taken to be the primary variables in three-dimensional beam formulations, see, e.g., [9–21] among many others who contributed to the topic. Authors use various parameterizations of finite rotations. Widely used are the three-parameter representations of which the often chosen "rotational vector" [22] is only one among many possibilities. The chosen parameterizations directly affect the discretization of rotations as thoroughly discussed in the textbooks [23] or [24] and also in articles [25–27]. The standard additive-type interpolation of three rotational parameters, despite often chosen due to its simplicity, is in contradiction with the configuration space of three-dimensional rotations. Crisfield and Jelenić [28,29] studied this problem in a systematic manner and proposed a new formulation based on the linked interpolation of incremental local rotations. An interesting alternative was proposed by Betsch and Steinmann [11] where all nine components of the director triad were interpolated. A comprehensive review and comparison of different interpolation strategies for rotational degrees of freedom were presented by Romero in [30] and in the recent paper by Bauchau and Han [31]. Rotations can also be represented using four parameters. Quaternions were introduced in beam formulations by Bottasso [32] and Kehrbaum and Maddocks [33], while McRobie and Lasenby [34] used Clifford algebra. Surprisingly, it was only recently that this ideas were successfully revived, see, e.g., [18,21,35–38]. Another interesting alternative is the fixed-pole approach by Bottasso and Borri [39], further exploited by Gaćeša and Jelenić [2], and the use of special Euclidean group to describe the configuration space proposed recently by Sonneville et al. [8].

When studying the dynamics of non-linear beams the temporal discretization of rotations and rotation related quantities is highly important. Efficient models preserve the configuration space of rotations in time. From the pioneering work of Simo and Vu-Quoc [40] numerous time integration strategies for non-linear beams were proposed. In [40] classical Newmark time integration scheme in Euclidean space was properly adapted to the group of rotations. Their approach can be interpreted as a special case of numerical methods on Lie groups later developed in papers by Crouch and Grossman [41], Munthe-Kaas [42] and Zanna [43]. For more detailed review of Lie group methods consult the textbook by Hairer et al. [44]. Unfortunately, the Newmark scheme, despite its proper adaptation to configuration space of the beam, exhibits a dramatic loss of numerical stability in some problems [45]. As the answer to this phenomenon several energy preserving or energy decaying algorithms were proposed, e.g., [5,7,10,12,39,45–50]. A promising alternative are the Lie group variational integrators, see, e.g., Demoures et al. [1] and the references therein.

From the perspective of total mechanical energy the spatial and temporal derivatives of configuration variables are the natural quantities. Additionally, when properly expressed with respect to the moving bases, the measures for the rate of change of rotation become additive quantities. This was the motivation for the strain-based beam formulation for static analysis of three-dimensional beams [51], where the invariance of strain measures under rigid-body motion was preserved by simply choosing the strain measures as the only interpolated variables. Despite the excellent performance of the strain-based formulation in statics, its extension to dynamics [52] exhibits similar problems as reported in [45] and numerical damping was needed to obtain the computational stability of long-term calculations. It is important to observe that angular velocities in dynamics have a similar role as the rotational strains in statics.

The idea exploited in this paper is to employ velocities and angular velocities as the primary unknowns of both spatial and temporal discretization. As the strain measures, velocities and angular velocities are mutually dependent [53,54], a special care is devoted to consistent discretization of strains. The proposed discretization preserves the kinematic compatibility of derivatives of rotations. The novel finite element discretization proposed here is based on interpolation of velocities, expressed in fixed basis, and angular velocities in a moving basis description, which also become the primary unknowns of the implicit time integration scheme. Two important computational benefits follow directly form this choice: (i) the additivity of primary unknowns enables the use of standard interpolation techniques in space and simplifies the computation of solution in iterative procedure at each time step; (ii) the strategy of choosing the time derivatives of configuration variables as the iterative quantities in implicit time integration scheme was found advantageous for stiff problems, see Hosea and Shampine [55]. To describe the rotations of the cross sections the rotational quaternions are used. The computational advantages of the quaternion representation of rotations are preserved, but additionally with the replacement of the primary unknowns we gain the considerable increase of numerical stability and robustness of the model without any other measures needed. The accuracy and favourable numerical performance of the proposed approach will be demonstrated by several examples.