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Non-linear vibration analysis of laminated composite plates resting on non-linear elastic foundations

T. Pirbodaghi[a,*], M. Fesanghary[b], M.T. Ahmadian[a]

[a]*School of Mechanical Engineering, Sharif University of Technology, 11365-9567 Tehran, Iran*
[b]*Department of Mechanical Engineering, Louisiana State University, Baton, Rouge, LA 70803, USA*

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Abstract

In this study, the homotopy analysis method (*HAM*) is used to obtain an approximate analytical solution for geometrically non-linear vibrations of thin laminated composite plates resting on non-linear elastic foundations. Geometric non-linearity is considered using von Karman's strain–displacement relations. Then, the effects of the initial deflection, ply properties, aspect ratio of the plate and foundation parameters on the non-linear free vibration is studied. Comparison between the obtained results and those available in the literature demonstrates the potential of *HAM* for the analysis of such vibration problems, whose governing differential equations include the quadratic and cubic non-linear terms. This study shows that only a first-order approximation of the *HAM* leads to highly accurate solutions for this type of non-linear problems.

1. Introduction

Thin laminated composite plates are widely used in civil and mechanical engineering applications. In many cases, these plates are subjected to relatively large amplitude vibrations with respect to their thickness, which may lead to the material fatigue and structural damage. These phenomena become more significant around the natural frequencies of the structure. Therefore, the non-linear vibration analysis is essential for a reliable design.

*Corresponding author. Tel.: +98 21 66165503; fax: +98 21 66000021.
E-mail address: pirbodaghi@gmail.com (T. Pirbodaghi).

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Due to the complexity of the governing differential equations, it is difficult to obtain exact analytical solutions for non-linear vibration of plates. As far as we know, researchers have concentrated on approximate analytical [1–5] and numerical techniques [6–13]. But analytical solutions appear more appealing than numerical solutions, since the analytical solutions yield an insightful understanding of the effect of system parameters and initial conditions and give a reference frame for verification and validation of other numerical approaches.

Traditional approximate analytical methods which have been widely used for weakly non-linear equations include perturbation methods such as the Lindstedt–Poincare, multiple time scales methods and the generalized averaging method of Krylov–Bogoliubov–Mitropolski [14–16]. The other available analytical methods, which have been used for solving strongly non-linear differential equations, are the harmonic balance [17–19], equivalent Linearization [20], describing function [21] and power series method [22–23].

The aforementioned traditional methods have their own limitations and involve tedious derivations and extensive computations. For example, the effectiveness of perturbation methods is limited to the weakly non-linear differential equations. As another example, the power series is used to determine analytical approximations to the periodic solutions of differential equations. Although this method is powerful and has been employed with some success, the method requires the generation of a coefficient for each term in the series. So, it is relatively time consuming and difficult to demonstrate the convergence of series [22,23].

The *HAM* [25–27] is an easy-to-use analytic tool for solving strongly non-linear differential equations. Compared to other analytical methods, the results of *HAM* converge quickly and require less computational effort with good accuracy. The effectiveness of *HAM* has been demonstrated in the analysis of some non-linear problems [24–29].

The main objective of this study is to use the homotopy analysis method for obtaining approximate analytical solutions for geometrically non-linear vibrations of thin laminated composite plates. And it is showed that the most significant feature of this method is its excellent accuracy for the whole range of oscillation amplitude values and initial conditions, while the traditional analytical techniques (e.g. perturbation method) lose their reliability and accuracy at higher vibration amplitudes and initial conditions.

2. Theoretical formulation

Consider a laminated composite plate with length a, width b and total thickness h as shown in Fig. 1. It is assumed that all layers are perfectly bonded with no slippage, and the reference surface is taken as the geometric mid-plane. The origin of the Cartesian coordinate system is located in one corner of the mid-plane (x–y plane) with the z-axis perpendicular to this plane.

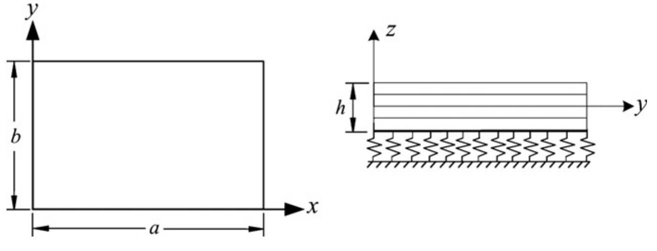

Fig. 1. Geometry of a thin laminated composite plate.