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A mortar method based on NURBS for curved interfaces

Horacio Florez[a,*], Mary F. Wheeler[b]

[a] The University of Texas at El Paso, El Paso, TX 79968, USA
[b] The Center for Subsurface Modeling, The University of Texas at Austin, TX 78712, USA

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Abstract

To tackle general sub-domain problems in geomechanics, we present an MFEM scheme on curved interfaces based on NURBS curves and surfaces. The goal is to have a more robust geometrical representation for mortar spaces, which allows gluing non-conforming interfaces on realistic geometries. The resulting mortar saddle-point problem is decoupled using standard domain decomposition techniques such as Dirichlet–Neumann, to exploit current parallel machine architectures. Two- and three-dimensional examples ranging from near-wellbore applications to field level subsidence computations show that the proposed scheme can handle problems of practical interest.

Keywords: Mortar finite element; NURBS; Domain decomposition; Dirichlet–Neumann

1. Background

Hydrocarbon production or the injection of fluids in a reservoir can produce changes in the rock stresses and in-situ geomechanics, potentially leading to compaction and subsidence with harmful effects in wells, cap-rock, faults, and the surrounding environment as well. Accurate simulations are required in order to predict these changes and their impact. In many cases, the flow simulation needs to be coupled to geomechanics, causing a significant increase in CPU time and memory requirements [1–3]. Poroelasticity is the basic theory to predict the compaction of a producing hydrocarbon reservoir and the related hazards, including land subsidence and borehole damage [4,5]. In terms of coupling, several approaches are possible, the most common being loose or iterative coupling methods in which the two problems are solved in sequence [3,6]. The monolithic approach, where all field equations are solved simultaneously, is regarded as the most suitable one [7,8] for this type of problems [9], but it is quite complicated to apply whenever the domains for flow and mechanics are not the same.

One possible way to mitigate the CPU burden associated with geomechanics calculations is the use of domain decomposition (DD) methods. DD entails the splitting of the domain into smaller sub-problems, while enforcing physically driven matching conditions at the interfaces.

* Corresponding author.
E-mail address: florezg@gmail.com (H. Florez).

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The Mortar Finite Element Method (MFEM) has been demonstrated to be a powerful technique aimed to formulate a weak continuity condition at the interface of subdomains in which different meshes, i.e. non-conforming or hybrid, and/or variational approximations are used. This method is particularly suitable when coupling different physics on different domains, such as elasticity and poroelasticity, for example, in the context of coupled flow and geomechanics.

In this area, geometrical aspects play an important role, as it can be impractical from the computational standpoint, to enforce the same mesh for flow and mechanics. Non-conforming discretizations are a highly attractive option for these problems since they provide considerable computational flexibility [10,11]. Bernardi, Maday and Patera introduced the MFEM for the Poisson equation [12] in order to formulate a weak continuity condition at the interface of subdomains in which different variational approximations are used. Relaxing the constraint on the boundaries of the interfaces, the formulation of Belgacem [13] with Lagrange multipliers is the standard framework in which the method is understood at present time. One of the key aspects of the method consists of defining appropriate spaces of Lagrange multipliers for enforcing the gluing constraint [14]. Additional references dealing with mortar methods can be found elsewhere [15–22]. Many specific applications to linear isotropic elasticity and elliptic problems can be found in the literature [16,17,14,23,24]. Flemisch et al. developed a dual mortar method for curved interfaces with applications to 2-D elasticity. Their method improves the performance of dual mortar when applied to curved surfaces in solid mechanics [16]. The dual mortar methods [22] are efficient due to the simple elimination of the Lagrange multipliers via the Schur decomposition. Also a comparison between the Nitsche method for linear elasticity with the mortar method using dual Lagrange multiplier spaces can be found in [25]. Girault et al. presented a multiscale domain decomposition method for solving linear elasticity where mortars are introduced at the interfaces as displacement boundary conditions [26]. Convergence of mortar methods applied to linear elasticity is analyzed in [27], this paper also presents a complete historical development of the mortar method in the context of domain decomposition. An inexact Dirichlet–Neumann nonconforming domain decomposition method applied to linear elasticity is presented in [28]. This method employs a mortar formulation based on dual basis functions and a special multi-grid method. Another recent work investigates several original algebraic techniques of approximation of the Dirichlet-to-Neumann map to absorbing boundary conditions in linear elasticity [29]. A recent textbook covering overlapping and non-overlapping domain decomposition schemes is presented in [30]. Implementation details, as well as the Neumann–Neumann and FETI algorithms, are well covered therein.

In this paper, we propose an extension to the MFEM that considers curved interfaces described by means of NURBS curves and surfaces. Most of these interfaces are obtained by interpolating a series of discrete points in all numerical examples included here. Subdomain meshes must honor these curves as constraints, which implies that by construction all mesh nodal points on the interface actually lie on the curves or surfaces. This condition is important in order to guarantee that a mortar projector can be computed in a straightforward manner.

The appropriate representation and meshing of the computational domain for the physical problem under study are necessary premises for a satisfactory simulation. In fact, one of the most demanding computational tasks in a simulation is defining the geometry because it will impact many aspects of the study such as the grid generation process [31]. Therefore, special methods must be applied to fit discrete data without sudden changes in curvature. The approach should be free of inflection points and, at minimum, it must enforce continuity C^2 of the fitted curve. In this work, this goal is achieved by using Bèzier, B-Spline, and NURBS curves and surfaces [32,33].

The paper is structured as follows: Section 2 describes the mathematical model for linear isotropic poroelasticity while Sections 3 and 4 tackle interpolation with NURBS curves and surfaces respectively. Section 5 is devoted to MFEM itself, and then Section 6 introduces decoupling techniques using domain decomposition methods. Section 7 briefly discusses mortar mappings, which are necessary to incorporate MFEM to existing parallel codes. Section 8 includes numerical examples in geomechanics, and the three last sections present concluding remarks, future work and acknowledgments respectively.

2. Mathematical model

This section discusses the governing equations for linear homogeneous isotropic poro-elasticity and their finite element formulation. We omit details for the sake of brevity, a more detailed treatment can be found in [34,10,35,36,1,6]. We consider a bounded domain $\Omega \subset \mathbb{R}^n$, $n = 2, 3$ and its boundary is $\Gamma = \partial\Omega$. Let $\mathcal{T}_h$ be a non-degenerate, quasi-uniform conforming partition of Ω composed of triangles or quadrilaterals for two-dimensional problems, and hexahedra or tetrahedra for three-dimensional problems. It can be shown that [2], for deformable porous media, the