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A new three-dimensional heat flux–temperature integral relationship for half-space transient diffusion

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Abstract

This paper derives a new integral relationship between heat flux and temperature in a transient, three-dimensional heat conducting Cartesian half space ($x>0$, $y\in(-\infty,\infty)$, $z\in(-\infty,\infty)$). A unified mathematical treatment has been developed based on operational and transform methods; and singular integral equation regularization. Regularization is accomplished based on a series of observations involving the diffusive nature of the operator. This newly developed relationship provides the local heat flux perpendicular to the front surface at any location within the half space. This expression suggests that an embedded plane of temperature sensors parallel to the surface can be used to acquire the local, in-depth heat flux in the x-direction. The relationship does not require a priori knowledge of the surface boundary condition which has analytically been removed in the process. The ill-posed nature of diffusion is highlighted owing to the appearance of the heating/cooling rate (°C/s) in the integrand of the new relationship. Integral relationships of this type are highly useful for experimental investigations since the in-depth heat flux can be extracted from well-established temperature transducers.

1. Introduction

Fourier's law represents a constitutive relationship between heat flux and temperature, i.e., in particular the temperature gradient. This particular law was historically devised at steady state conditions and is known to possess an unreal physical feature; namely, infinite

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Nomenclature

C	heat capacity, kJ/(kg °C)
k	thermal conductivity, W/(m °C)
q''_x	dimensional heat flux in x-direction, W/m^2
q''_y	dimensional heat flux in y-direction, W/m^2
q''_z	dimensional heat flux in z-direction, W/m^2
t	time, s
t_0	dummy time variable, s
T	temperature, °C
T_0	initial temperature, °C
u	dummy time variable, s
x	spatial variable, m
x_0	dummy spatial variable, m
y	spatial variable, m
y_0	dummy spatial variable, m
z	spatial variable, m
z_0	dummy spatial variable, m

Greek

α	thermal diffusivity, $k/(\rho C)$, m^2/s
ρ	density, kg/m^3

thermal propagation speed. For most common physical problems, this paradox is overlooked. Commercial heat flux gauges are designed based on $q''_x \approx -k\Delta T/\Delta x$, where q''_x is the heat flux in the x-direction, T is the temperature, Δx is the distance between thermocouples, and k is the thermal conductivity. Heat flux gauges are normally surface mounted owing to their size and construction. However, it is possible to acquire the heat flux in an alternative manner using the time domain. This is possible if the general law (first law of thermodynamics) and constitutive equation for heat flux (Fourier's law) are combined to obtain the classical heat equation in the temperature variable. Frankel and his colleagues [1–7] have developed a unified mathematical treatment for obtaining these new integral relationships that do not involve any knowledge of spatial gradients. Hence, only time histories of temperature are required. This offers some hope for estimating in-depth heat fluxes using in-depth temperature sensors. Laboratory testing for benchmarking purposes is normally performed in one and two dimensions. As part of the analytical development process, it is also well known that experimental studies for estimating thermophysical properties are normally conceived in reduced dimensions. Hence, one- and/or two-dimensional experimental studies are commonly performed. As such, Frankel and his colleagues have primarily focused on developing one- and two-dimensional relationships for isotropic, orthotropic and anisotropic materials [7] that account for noisy data [1,4]. A scheme based on Gauss digital filtering the acquired temperature data have been successfully demonstrated and implemented using both numerical data with random noise [1,4] and real experimental data.