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A fictitious domain/distributed Lagrange multiplier based fluid–structure interaction scheme with hierarchical B-Spline grids

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Highlights

- We present a new numerical scheme for fluid-structure interaction based on B-Spline cartesian grids. The fluid grid near the immersed solids can be refined locally using hierarchical B-Splines.
- The solid is assumed to be infinitely thin strcture and is modelled using geometrically-exact beam formulation.
- A second-order and unconditionally stable generalised-α scheme is used for time integration of both the fluid and solid domains. This allows us use of larger time increments without losing accuracy.
- The proposed scheme has been applied to several fluid-flexible body interaction examples involving moderate and large deformations of the structure as well as simulating dynamically complex bi-stable phenomenon observed in filaments and swimming organisms.

Abstract

We present a numerical scheme for fluid–structure interaction based on hierarchical B-Spline grids and fictitious domain/distributed Lagrange multipliers. The incompressible Navier–Stokes equations are solved over a Cartesian grid discretised with B-Splines. The fluid grid near the immersed solids is refined locally using hierarchical B-Splines. The immersed solid is modelled as geometrically-exact beam discretised with standard linear Lagrange shape functions. The kinematic constraint at the fluid–solid interface is enforced with distributed Lagrange multipliers. The unconditionally-stable and second-order accurate generalised-α method is used for integration in time for both the fluid and solid domains. A fully-implicit and fully-coupled solution scheme is developed by using the Newton–Raphson method to solve the non-linear system of equations obtained with Galerkin weak formulation. First, the spatial and temporal convergence of the proposed scheme is assessed by studying steady and unsteady flow past a fixed cylinder. Then, the scheme is applied to several benchmark problems to demonstrate the efficiency and robustness of the proposed scheme. The results obtained with the present scheme are compared with the reference values.

Keywords: Fictitious domain methods; Immersed boundary methods; Fluid–structure interaction; Hierarchical B-Splines; Distributed Lagrange multipliers; Flow-induced vibrations

1. Introduction

Fluid–structure interaction is a phenomenon frequently encountered in the fields of science and engineering. Many factors, such as properties of the fluid and structure, extent of deformations of the structure, and instabilities due

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to added-mass, influence the development and applicability of a numerical scheme for simulating FSI problems. In the traditional arbitrary Lagrangian–Eulerian (ALE) approach with body-fitted meshes, which is extensively studied and understood (see Chapter 14 in [1] and references therein), the fluid is solved on a body-fitted mesh which is either adjusted or re-meshed depending upon the extent of the deformations of the solid, see [2,3]. However, ALE comes with several disadvantages: (a) it requires the generation of body-fitted meshes which is a cumbersome task for complicated geometries, (b) the fluid mesh needs to be updated or re-meshed depending upon the extent of solid deformations, (c) every re-meshing step involves a data-mapping from old mesh to the new mesh which is also prone to errors. Hence, the applicability of ALE formulation is limited by the ease of generating body-fitted meshes and the robustness of re-meshing algorithms. Moreover, for more demanding fluid–structure interaction applications involving topological changes, e.g., self-contacts in structural model, simulation of check-valves and multiple fibres submerged in flow, ALE formulation may fail because of zero-volume elements when the structural parts are in contact. Extending such numerical schemes for FSI based on body-fitted meshes to 3D problems where the solid undergoes extreme deformations is a challenging task, for which it is difficult to ensure the robustness of the scheme. To overcome these difficulties alternate solution strategies based on fixed Cartesian grids are being explored.

The immersed or embedded or non-body-fitted or Cartesian grid based methods are simpler, easy to implement and computationally more efficient than the methods based on body-fitted meshes for problems where the solids undergo huge deformations and/or topological changes and multiphase and mixing flows. In these type of methods the fluid is modelled in an Eulerian frame of reference and the solid is modelled in a Lagrangian frame. The solid, that may either be fixed or undergoing extreme deformations and/or topological changes, is immersed into the fluid grid with discretisation that does not need to match the solid boundaries. The interface conditions at the fluid–solid interface are enforced via several techniques and it is this technique that distinguishes different immersed methods. To our knowledge, immersed boundary methods (IBM) introduced and pioneered by Peskin [4] are the first research work carried out in the direction of non-body-fitted meshes. In [4] and its variation [5–8] the kinematic constraint at fluid–solid interface is enforced using body-force approach. The body-force is computed assuming that the Lagrange points are connected to artificial springs with high stiffness values. This method restricts the time steps to small values irrespective of whether the fluid solver is implicit or explicit. Later, immersed interface method (IIM) was introduced by [9–12] in which derivatives in the cells cut the boundary of the immersed solid are modified in order to accommodate the jumps in velocity and/or pressure. Due to this modification process IIM is applicable only to FSI problems with bulky solids. Historically, in majority of the research work carried out with IBM and IIM the fluid problem is solved using finite-difference and finite-volume grids which lack local refinement capability.

IBM and IIM based on standard finite element meshes are studied in [13,14]. Zhang and Gay [15], Yao et al. [16] and Zhang et al. [17] studied immersed finite element methods for fluid–structure interaction problems. However, the amount of research in such methods is limited and most of these methods still inherit the disadvantages of Peskin's immersed boundary method [4]. For example, the way the interacting forces are computed and velocities are interpolated from fluid mesh to solid mesh and vice-versa restricts the time steps to very small values.

Höllig [18,19] used B-Splines for the first time in the context of immersed finite element methods and developed weighted extended B-Splines (WEB-Splines). Later, this concept was extended by [20–22] to FSI problems. Though this method seems to be promising to simulate FSI problems, it involves a basis function modification algorithm in order to tackle instabilities due to the presence of small cut-cells. Also, for problems involving thin structures this approach poses several difficulties in identifying and modifying the basis functions. To overcome these limitations, the fictitious domain method (FDM) pioneered by Glowinski [23–31] seems to be an efficient alternative. FDM is another class of embedded methods where the kinematic constraint at the fluid–solid interface is enforced using Lagrange multipliers. FDM offers several advantages over the classical IBM [4], IIM [9] and WEB-Spline method [18]. While the kinematic constraint at the fluid–solid interface is applied weakly in IBM, it is applied strongly in FDM using Lagrange multipliers. Moreover, the Lagrange multipliers are tractions on the boundary of the immersed body which can be used directly for FSI problems. Furthermore, in FDM, there is no need to modify the basis functions of the fluid grid in order to ensure the cut-cell stabilisation as the fluid is solved everywhere in background fluid grid. So far, in the literature, the fluid grid in FDM is discretised with the standard Lagrange polynomials – Taylor–Hood or Crouziex–Raviart family elements [32] – with or without bubble functions [33]. In this work we propose a fictitious domain formulation for simulating FSI problems based on hierarchical B-Spline grid. The numerical scheme proposed in this paper can be considered as an extension of immersogeometric framework described by [34] in the sense that the non-uniform rational B-splines (NURBS) used in [30] to discretise the background fluid grid are replaced here with