Available online at www.sciencedirect.com
ScienceDirect
Journal of the Franklin Institute 347 (2010) 116–129
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

On the castor dynamic behavior

D. de Falco[a], G. Di Massa[b,*], S. Pagano[b]

[a]Seconda Universitá degli Studi di Napoli, Dipartimento di Ingegneria Aerospaziale e Meccanica, Napoli, Italy
[b]Universitá degli Studi di Napoli ''Federico II'', Dipartimento di Ingegneria Meccanica per l'Energetica, Via Claudio 21, 80125 Napoli, Italy

Received 30 July 2009; received in revised form 12 October 2009; accepted 12 October 2009

Abstract

In this paper, a model describing the castor three-dimensional dynamic behavior, is presented. A procedure, based on the Udwadia–Kalaba formulation has been implemented to write the castor non-linear motion equations. The model takes into account the flexural and lateral compliance of the castor frame and the phenomena related to the wheel rotation. To prescind from the wheel–road interaction parameters, the hypothesis of no sideslip of the wheel has been adopted but other tire models can be easily implemented. The non-linear system stability is evaluated integrating the motion equations and performing a fitting procedure on the natural steering oscillation.

Keywords: Castor; Shimmy; Stability; Udwadia–Kalaba formulation

1. Introduction

The term ''castor'' is used to define any swivel-wheel whose contact patch area with the ground lies behind the intersection of the steering axis with the ground. The study of castor oscillations regards many applications (even if there are some differences) ranging from aircraft landing gear ''shimmy'' to motorcycle ''wobble'' and to the ''waving'' of towed vehicles. It consists of rapid oscillations of the whole castor wheel assembly about its king pin. In some operational conditions, the oscillations can reach great amplitudes resulting in an increase in rolling resistance and tire wear; moreover, in these conditions, the castor components are stressed by dangerous phenomena of fatigue and wear.

*Corresponding author.
E-mail address: gdimassa@unina.it (G. Di Massa).

0016-0032/$32.00
doi:10.1016/j.jfranklin.2009.10.013

Nomenclature

l_1	distance between the castor center of mass G and wheel–ground contact point O
l_2	distance between castor center of mass G and steering axis
G	castor center of mass
g	gravity acceleration
I_a	castor trailing arm inertia matrix
I_w	wheel inertia matrix
K_ψ	camber frame stiffness
K_y	trailing arm lateral stiffness
d	wheel offset
h	trailing arm height
m_a	trailing arm mass
m_w	wheel mass
m	whole castor mass
P	wheel–road contact point
R	wheel radius
t	caster trail
T	kinetic energy
v	forward speed
V	potential energy
y	lateral displacement
ε	rake angle
δ	caster rotation around steer axis
ρ	whole castor inertia radius
σ	anti-shimmy damping
ψ	camber angle
$[\,]^+$	pseudo-inverse matrix
$\zeta, \omega_s, \omega_n$	under-damped harmonic function parameters

The first scientific papers on the shimmy phenomenon were published about 80 years ago but many details on the phenomenon mechanism are not yet fully understood since it depends on a great number of factors such as overall vehicle dynamics, compliance of the castor components, backlashes and tire characteristics [1–5].

One of the most common castor wheels is that used for supermarket trolleys, office chairs, etc. (Fig. 1a); the wheel is auto-aligning with respect to the motion direction as it can steer around a vertical king pin whose axis is advanced with respect to the wheel axis. The general castor scheme (Fig. 1b) is characterized by a steering axis tilted by an angle (caster angle) with respect to the vertical direction, related to the wheel radius R, the trail t and the wheel offset d by the following expression:

$$t = Rtg\varepsilon - \frac{d}{\cos\varepsilon}. \quad (1)$$