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Generalization of all stabilizing compensators for finite-dimensional linear systems

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Abstract

For finite-dimensional linear systems, the Youla–Kucera parameterization (YKP) with a Q parameter over RH_∞ is assumed to satisfy the Diophantine identity. However, the stability is guaranteed if the Diophantine equation is the "$\mathbf{U}(RH_\infty)$" equality, but not if it is the "identity" equality. However, Vidyasagar's structure with an H parameter over $\mathbf{U}(RH_\infty)$ is an observer–controller configuration that satisfies the Diophantine equation. This study discusses the deficiency of the Diophantine identity; expands the YKP using an H parameter over $\mathbf{U}(RH_\infty)$, and expands the Vidyasagar's structure using a Q_v parameter over RH_∞ so that both of the expanded parameterizations satisfy the Diophantine equation and are equivalent for all stabilizing compensators. Moreover, an equation that relates to Q, Q_v, and H will be introduced to establish relationships among the YKP, Vidyasagar's structure and both expanded parameterizations.

Keywords: Youla–Kucera parameterization; Diophantine equation; Diophantine identity; All stabilizing compensators

1. Introduction

All stabilizing controllers have to date involve well-known Youla–Kucera parameterization (YKP) with a Q parameter over RH_∞ for finite-dimensional linear systems [1–6]. The YKP is supposed to satisfy the Bezout identity or the Diophantine identity [3,6]. However, the stability is guaranteed when the Diophantine equation is the "$\mathbf{U}(RH_\infty)$"

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equality rather than the "identity" equality. The assumption of the Diophantine identity suggested by Youla et al. [3] causes the "$\mathbf{U}(RH_\infty)$" parameter to be omitted from the parameterization of all stabilizing compensators. The omission of the "$\mathbf{U}(RH_\infty)$" parameter makes finding another form of parameterization difficult. Vidyasagar [6] agreed that the YKP were all stabilizing compensators, and so included the stabilizing solutions to Vidyasagar's structure (VS) that he proposed. The VS with an H parameter over $\mathbf{U}(RH_\infty)$ is an observer–controller configuration that satisfies the Diophantine equation. However, the solutions to VS differ greatly from those to the YKP, as will be discussed herein. YKP and VS motivate the present study of the deficiency with the Diophantine identity, the expansion of the YKP (EYKP) by adding an H parameter over $\mathbf{U}(RH_\infty)$, and the expansion of VS (EVS) by adding a Q_v parameter over RH_∞ so that both of the expansions satisfy the Diophantine equation and are equivalent to all stabilizing compensators. Moreover, since H builds the bridge between the EYKP and the EVS, an equation in Q, Q_v, and H that connects YKP, VS, EYKP, and EVS can be introduced.

Other parameterizations of stabilizing controllers for other systems are available [7–11]. Infinite-dimensional linear systems [7,8], admit (weakly) left/right/ doubly coprime factorizations by means of Banach algebras; such systems which are algebraically and topologically more complex than the ring RH_∞. For structurally stable multidimensional systems, it is not known yet whether or not a stabilizable plant always has its doubly coprime factorization [9,10]; parameterization method of stabilizing controller without doubly coprime factorization is presented [11].

The study is laid out as follows. Section 2 discusses the deficiency of the Diophantine identity, which reduces the solutions to all stabilizing compensators. Moreover, the EYKP is introduced. Section 3 investigates VS again and develops the EVS. Moreover, the relationships among the EVS, VS, the EYKP, and the YKP are specified by an equation that relates Q, Q_v, and H. Section 4 presents an example that confirms the accuracy of the derivation and demonstrates the selection of Q, Q_v, and H and draws conclusions.

Mathematical notations:

$\mathbf{Re}(s)$	real part of s, where s is a complex variable
RH_∞	set of analytic real rational function matrices analytic in $\mathbf{Re}(s)>0$
$\boldsymbol{R}(s)$	set of real rational functions
$\boldsymbol{R}^{n\times n}$	consists of $n\times m$ matrices whose elements are in $\boldsymbol{R}(s)$
$H\in\boldsymbol{R}^{n\times n}$	unimodular matrix over RH_∞ if the inverse of H exists over RH_∞
$\mathbf{U}(RH_\infty)$	set of unimodular matrices over RH_∞
$\boldsymbol{R}_{\mathrm{p}}(s)$	set of proper rational transfer function matrices

2. The deficiency of diophantine identity

This section discusses the deficiency of the Diophantine identity, which the YKP satisfies, and elaborates an output-feedback controller and an observer–controller compensator with two independent parameters. One parameter is over RH_∞ and the other is over $\mathbf{U}(RH_\infty)$. Both of the compensators are input–output equivalent, and can be transformed into "the EYKP", which satisfies the Diophantine equation with an extra property that the YKP does not have.

The following well-known facts are presented as they will be extensively applied herein.