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Bifurcation analysis and control of a discrete harvested prey–predator system with Beddington–DeAngelis functional response

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Abstract

In this paper, we study a discrete prey–predator system with harvesting of both species and Beddington–DeAngelis functional response. By using the center manifold theorem and bifurcation theory, we establish that the system undergoes flip bifurcation and Hopf bifurcation when the harvesting effort of prey population passes some critical values. Numerical simulations exhibit period-6, 10, 12, 14, 20 orbits, cascade of period-doubling bifurcation in period-2, 4, 8, 16 orbits and chaotic sets. At the same time, the numerically computed Lyapunov exponents confirm the complex dynamical behaviors. Moreover, a state delayed feedback control method, which can be implemented only by adjusting the harvesting effort for the prey population, is proposed to drive the discrete prey–predator system to a steady state.

Keywords: Beddington–DeAngelis functional response; Flip bifurcation; Hopf bifurcation; State delayed feedback control; Chaos; Lyapunov exponents

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1. Introduction

At present, mankind is facing the dual problems of resource shortages and environmental degradation. This has spawned a rapidly growing interest in the analysis and modeling of biological systems. The exploitation and harvesting of biological resources for human needs have occurred in many fields, such as fishery, wildlife, forestry management and so on. In many earlier studies [1–4], it has been shown that harvesting has a strong impact on population dynamics, ranging from rapid depletion to complete preservation of biological populations.

Many scholars have carried out the study of population dynamics with various functional responses, such as the Monod-type [4–6], Holling-type [7–11], Ivlev-type [12,13] and so on. In addition, the prey–predator model with the Beddington–DeAngelis functional response is also well-known. This model can avoid some of the singular behaviors of ratio-dependent models at low densities and provide a better description of predator feeding over a range of prey–predator abundances. The following differential equations describe the prey–predator model with Beddington–DeAngelis functional response

$$\begin{cases} \dfrac{d\tilde{x}}{d\tilde{t}} = r\left(1-\dfrac{\tilde{x}}{K}\right)\tilde{x}-\dfrac{d_1\tilde{x}\tilde{y}}{a+b\tilde{x}+c\tilde{y}}, \\ \dfrac{d\tilde{y}}{d\tilde{t}} = \left(-\mu+\dfrac{ed_1\tilde{x}}{a+b\tilde{x}+c\tilde{y}}\right)\tilde{y}, \end{cases} \tag{1}$$

where $\tilde{x}$ and $\tilde{y}$ represent the prey density and predator density at time $\tilde{t}$, respectively; the predator consumes the prey with Beddingron–DeAngelis function response $d_1\tilde{x}\tilde{y}/(a+b\tilde{x}+c\tilde{y})$ and contributes to its growth rate $ed_1\tilde{x}\tilde{y}/(a+b\tilde{x}+c\tilde{y})$. $r>0$ is the intrinsic growth rate of prey, $K>0$ is the carrying capacity of prey and $\mu>0$ is the death rate of predators in the absence of food.

With the idea of harvesting, we consider the following prey–predator system:

$$\begin{cases} \dfrac{d\tilde{x}}{d\tilde{t}} = r\left(1-\dfrac{\tilde{x}}{K}\right)\tilde{x}-\dfrac{d_1\tilde{x}\tilde{y}}{a+b\tilde{x}+c\tilde{y}}-q_1H_1\tilde{x}, \\ \dfrac{d\tilde{y}}{d\tilde{t}} = \left(-\mu+\dfrac{ed_1\tilde{x}}{a+b\tilde{x}+c\tilde{y}}\right)\tilde{y}-q_2H_2\tilde{y}, \end{cases} \tag{2}$$

where $q_1H_1\tilde{x}$ and $q_2H_2\tilde{y}$ are the catch rate functions. $q_i>0$ and H_i, $i=1,2$ are the catchability coefficients and the harvesting efforts, respectively. Assume $H_1 \in R$ and $H_2>0$, which can be interpreted as reintroducing or harvesting prey species and keeping predator ones caught, respectively.

We nondimensionalize the system (2) with the following scaling:

$$u=\frac{\tilde{x}}{K}, \quad v=\frac{d_1}{ra}\tilde{y}, \quad t=r\tilde{t},$$

and then obtain the form

$$\begin{cases} \dfrac{du}{dt} = (1-u)u-\dfrac{uv}{1+a_1u+a_2v}-E_1u, \\ \dfrac{dv}{dt} = \left(-d+\dfrac{\beta u}{1+a_1u+a_2v}\right)v-E_2v, \end{cases} \tag{3}$$