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# Hierarchical NURBS and a higher-order phase-field approach to fracture for finite-deformation contact problems

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## Highlights

- We present a hierarchical refinement scheme for isogeometric analysis.
- A frictional Mortar contact algorithm for hierarchical refined NURBS is introduced.
- A higher-order phase-field approach to brittle fracture is applied to the concept of HNURBS and combined with frictional contact problems.

## Abstract

In this paper we investigate variationally consistent Mortar contact algorithms applied to a phase-field approach to brittle fracture. Phase-field approaches allow for an efficient simulation of complex fracture problems, as they arise in contact and impact situations. To improve accuracy and convergence, a fourth-order phase-field model is considered, requiring $C^1$ continuity throughout the domain. An isogeometrical framework is used for the spatial discretisation subject to hierarchical refinements to resolve local features. This reduces the computational effort tremendously, as will be shown in a series of representative examples.


*Keywords:* Hierarchical refinement; NURBS; Phase field; Mortar contact; Finite deformations; Fracture

## 1. Introduction

A phase-field approach to fracture is a most general methodology to predict failure mechanisms in solids. In contrast to the costly and complex computational modelling of sharp cracks, a diffusive interface is introduced, see Miehe et al. [1,2] and Kuhn and Müller [3]. The phase-field itself is described by an order parameter $\mathfrak{s}$ that is driven by a corresponding partial differential equation, see Weinberg and Hesch [4] for a detailed investigation on Allen–Cahn type as well as Cahn–Hilliard type equations. It is assumed that the material fails locally upon the attainment of a specific

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fracture energy or critical energy-release rate, as introduced by Francfort and Marigo [5] and Bourdin et al. [6]. This allows to formulate a variational statement for brittle fracture, see, among many other Karma et al. [7,8] and Henry et al. [9,10]. Applications to ductile fracture have been recently proposed in Miehe et al. [11], whereas phase-field models for cohesive fracture have been addressed in Verhoosel and de Borst [12]. An extension to large deformations relying on a multiplicative decomposition of the deformation gradient into a compressive and a tensile part along with a structure preserving time integration scheme is given in Hesch and Weinberg [13].

The finite element discretisation of a phase-field approach requires high spatial resolution of the diffusive interface, since the element size $h \ll l$, where $l$ represents a specific length parameter of the phase-field approach. To reduce the computational effort, a higher-order phase-field approach as proposed by Borden et al. [14] can be applied to the phase-field, improving the accuracy and convergence of the numerical solution. This fourth-order partial differential equation cannot be discretised by standard Lagrangian shape functions. Thus, we make use of non-uniform rational B-splines (NURBS) in the context of Isogeometric Analysis for the spatial discretisation, see Cottrell et al. [15] for a comprehensive review on this topic. NURBS basis functions allow us to predefine the basis functions continuity within their construction, which makes them ideal for the treatment of higher-order problems.

Although NURBS basis functions have local support, they are not restricted to a single finite element. Moreover, in the multivariate case they have a tensor product structure. This is a major drawback for the introduction of local refinement procedures. T-splines have been introduced to break the tensor product structure of the spline base, see Bazilevs et al. [16] and Evans et al. [17]. Due to severe limitations of T-Splines (see Vuong et al. [18] for details), hierarchical refinement procedures have been developed, see Forsey and Bartels [19], Schillinger et al. [20] and Bornemann and Cirak [21], see also Borden et al. [22] for adaptive refinement in the context of phase-field models to brittle fracture. Hierarchical refinement procedures replace B-spline and NURBS basis functions on the refined level by a linear combination of scaled and copied versions of themselves, maintaining the required continuity, see Jiang and Dolbow [23] and Hesch et al. [24] for the application to general phase-field problems. In particular, we aim at a hierarchical refinement formulation maintaining the partition of unity, suitable to be adapted to traditional contact mechanical formulations, and equipped with additional features to account for repeated knots.

In Dittmann et al. [25], thermomechanical Mortar contact problems in the context of isogeometric analysis are addressed. This work analyses a thermodynamically consistent framework including the energy transfer between the mechanical and the thermal field due to friction and the variationally consistent description of the contact interface on the basis of Mortar methods, see de Lorenzis et al. [26,27] and Temizer [28,29] for further details on Mortar contact formulations for isogeometrical analysis. Hierarchical refinements for frictionless contact have been investigated in Temizer and Hesch [30]. In this work, these investigations are extended by incorporating friction within a dynamic framework and additionally applying the resulting algorithms to contact problems involving phase-field fracture.

An outline of the paper is as follows. The higher-order phase-field approach to fracture and the corresponding contact formulations are presented in Section 2. The spatial discretisation using hierarchical refinements for the NURBS basis functions as well as the Mortar formulation will be dealt with in Section 3. The temporal discretisation is outlined in Section 4, followed by representative numerical examples in Section 5. Eventually, conclusions are drawn in Section 6.

## 2. Governing equations

The description discussed in this section summarises the fundamental developments and features of a coupled multi-field problem for multiple bodies in contact. In addition to the mechanical field defined in its reference configuration of the bounded domain $\mathcal{B} \subset \mathbb{R}^n$, $n \in [2, 3]$, we assume the existence of a phase-field to characterise the diffusive crack modelling inside all bodies taken into account. Adopting a Lagrangian formulation, we introduce the deformation mapping

$$\boldsymbol{\varphi}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \to \mathbb{R}^n, \tag{1}$$

to map a material point $\boldsymbol{X}$ to its actual position at time $t \in \mathcal{I} = [0, T]$. Consistent with this Lagrangian formulation, the phase-field is described by an order parameter

$$\mathfrak{s}(\boldsymbol{X}, t) : \mathcal{B}_0 \times \mathcal{I} \to \mathbb{R}, \quad \mathfrak{s} \in [0, 1], \tag{2}$$