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Isogeometric dual mortar methods for computational contact mechanics

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Highlights

- A dual mortar method for NURBS-based isogeometric analysis is developed.
- Spatial convergence orders are analyzed for mesh tying and contact mechanics.
- A lack of reproduction properties limits the convergence in mesh tying applications.
- Optimal convergence results are achieved for contact applications.
- The higher smoothness of NURBS delivers smoother results for contact forces.

Abstract

In recent years, isogeometric analysis (IGA) has received great attention in many fields of computational mechanics research. Especially for computational contact mechanics, an exact and smooth surface representation is highly desirable. As a consequence, many well-known finite element methods and algorithms for contact mechanics have been transferred to IGA. In the present contribution, the so-called dual mortar method is investigated for both contact mechanics and classical domain decomposition using NURBS basis functions. In contrast to standard mortar methods, the use of dual basis functions for the Lagrange multiplier based on the mathematical concept of biorthogonality enables an easy elimination of the additional Lagrange multiplier degrees of freedom from the global system. This condensed system is smaller in size, and no longer of saddle point type but positive definite. A very simple and commonly used element-wise construction of the dual basis functions is directly transferred to the IGA case. The resulting Lagrange multiplier interpolation satisfies discrete inf–sup stability and biorthogonality, however, the reproduction order is limited to one. In the domain decomposition case, this results in a limitation of the spatial convergence order to $\mathcal{O}(h^{3/2})$ in the energy norm, whereas for unilateral contact, due to the lower regularity of the solution, optimal convergence rates are still met. Numerical examples are presented that illustrate these theoretical considerations on convergence rates and compare the newly developed isogeometric dual mortar contact formulation with its standard mortar counterpart as well as classical finite elements based on first and second order Lagrange polynomials.

Keywords: Isogeometric analysis; Contact mechanics; Mortar finite element methods; Dual Lagrange multipliers; Finite deformation

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1. Introduction

Robust and accurate contact discretizations for nonlinear finite element analysis have been an active field of research in the past decade and a new class of formulations emerged with the introduction of isogeometric analysis (IGA) [1]. IGA is intended to bridge the gap between computer aided design (CAD) and finite element analysis (FEA) by using the smooth non-uniform rational B-splines (NURBS) or T-splines common in CAD also as a basis for the numerical analysis. The use of such smooth basis functions has some advantages over classical Lagrange polynomials for FEA such as a possibly higher accuracy per degree of freedom [2,3] and, more importantly, a higher inter-element continuity. While finite elements based on Lagrange polynomials are limited to C^0 inter-element continuity independent of the polynomial order p, NURBS can be constructed with a maximum of C^{p-1} continuity. This high continuity results, amongst others, in a smooth surface representation which makes the application to computational contact mechanics particularly appealing, which has already been anticipated in the original proposition of IGA by Hughes et al. [1].

As a consequence, in the past five years various discretization techniques have been developed for IGA or transferred from finite element based contact mechanics to IGA, such as node-to-segment [4], Gauss-point-to-segment [5–10] and mortar methods [5,6,11–14]. We refer to the recent review in [15] for a comprehensive discussion of such methods, comparisons to their finite element counterparts and further references. In addition to the mentioned methods based on an isogeometric Galerkin approximation, the higher inter-element continuity of NURBS basis functions allows for the use of collocation methods, see [16] for a general introduction and [17,18] for an application to computational contact mechanics. Besides the discretization technique, computational contact algorithms can be distinguished with respect to the underlying solution procedure. While Gauss-point-to-segment approaches are, due to their lack of inf–sup stability (see e.g. [5,7] for numerical investigations), usually combined with a penalty approach, see [7–10], node-to-segment and mortar formulations can be combined with penalty methods [5,6], Uzawa-type algorithms [11], Lagrange multiplier methods [13,14] or augmented Lagrange methods [12]. In contrast to penalty methods, the other mentioned methods fulfill the contact constraints in a discrete sense exactly.

Proposing an alternative approximation of the Lagrange multiplier, so-called dual mortar methods were originally introduced in the context of domain decomposition [19] and later extended to small deformation frictionless contact in 2D [20] and 3D [21], small deformation frictional contact [22], finite deformation frictionless contact [23,24] and finite deformation frictional contact [25]. Since NURBS are naturally associated with higher order approximations, the extension of the dual mortar method to second order Lagrange finite elements in [26,27] including optimal a priori error estimates is also worth mentioning. A comprehensive review on dual mortar methods for contact mechanics can be found in [28,29]. In the context of domain decomposition in IGA, optimality and stability of standard mortar methods have only very recently been investigated in [30–33], where also the construction of dual B-spline basis functions has been outlined theoretically [33].

In this contribution, we present a mortar contact formulation for IGA using a dual Lagrange multiplier method where the inequality constraints are reformulated using nonlinear complementarity (NCP) functions. In contrast to standard mortar methods, the use of a dual Lagrange multiplier basis in mortar contact algorithms yields a localization of the contact constraints due to the biorthogonality property of the dual basis functions. Thus, they can be combined with efficient nonlinear solution schemes such as semi-smooth Newton methods as an active set strategy, and the additional Lagrange multiplier degrees of freedom can easily be condensed from the global system of equations. The aim of this work is to transfer the concept of dual basis functions from finite elements to IGA, thus combining the efficiency of dual mortar methods with the beneficial IGA inherent concept of a smooth geometrical representation. The newly developed isogeometric dual mortar method is applied to both domain decomposition and (unilateral) frictional contact problems with finite deformations. In both applications, spatial convergence orders for the discretization error using uniform mesh refinement are studied numerically. To the best of our knowledge, this contribution presents the first realization of a dual NURBS approach in both domain decomposition and contact mechanics, and probably also a first proper investigation on spatial convergence orders of higher order NURBS contact algorithms in general.

The remainder of this paper is organized as follows. Section 2 reviews the three-dimensional continuum mechanical description of a two-body frictional contact problem with finite deformations. Next, NURBS basis functions are briefly introduced in Section 3 as a general tool for isogeometric analysis. These NURBS basis functions are then used in Section 4 to discretize the contact problem. Most importantly, biorthogonal basis functions with the same support