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Dissipativity analysis and synthesis of a class of nonlinear systems with time-varying delays

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Abstract

In this paper, new results are established for the delay-independent and delay-dependent problems of dissipative analysis and state-feedback synthesis for a class of nonlinear systems with time-varying delays with polytopic uncertainties. This class consists of linear time-delay systems subject to nonlinear cone-bounded perturbations. Both delay-independent and delay-dependent dissipativity criteria are established as linear matrix inequality-based feasibility tests. The developed results in this paper for the nominal system encompass available results on $\mathscr{H}_\infty$ approach, passivity and positive realness for time-delay systems as special cases. All the sufficient stability conditions are cast. Robust dissipativity as well as dissipative state-feedback synthesis results are also derived. Numerical examples are provided to illustrate the theoretical developments.

Keywords: Dissaptivity; Dissipative analysis; Dissipative synthesis; Time-delay systems; Nonlinear systems; State feedback

1. Introduction

During the last decade, stability and stabilization of time-delay systems have been topics of recurring interest over the past decades since delays are often the main causes of instability and poor performance of dynamic systems. More importantly, time-delay

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models are encountered in various engineering and physical systems. These include, but not limited to, chemical processes, long transmission lines, communication networks, see [5,12,17] and their bibliographies. In the robust stability context, where the convex polytopic uncertainties and variation of the time-delay pattern are taken into account, development of stability and stabilization schemes are becoming especially important. Among the research topics relevant to this work is the passivity analysis and synthesis as applied to different classes of time-delay systems [8,9,16,18–20,27] using classical definitions of passivity and positive realness [1,7,10,14,15,23–26]. These results show that passivity-based methods are highly effective in producing robust controllers to classes of time-delay systems with norm-bounded parametric uncertainties. In particular, when the time-delay factor is unknown constant, it is emphasized that delay-dependent passivity yields less conservative performance results. Some recent passivity-based results for linear time-delay (LTD) systems are reported in [11,21]. Some related results to the present work can be found in [2–4,22] where alternative filtering and observation schemes are employed.

This paper develops new results for the problems of dissipative analysis and state-feedback synthesis for a class of nonlinear systems. This class is composed of linear system with time-varying delays and cone-bounded nonlinearities. Two cases of time-delay patterns are considered: unknown constants and continuous time-varying bounded functions. In this work, we first treat the unknown constant time-delay and develop delay-independent dissipative analysis. We show that all available results on $\mathscr{H}_\infty$ approach [10], passivity and positive realness for LTD systems as actually special cases of the present approach. For delay-dependent dissipative analysis, we employed the descriptor approach as a method of exhibiting the delay-dependent dynamics. It is then established that the resulting dissipativity conditions can be cast in a linear matrix inequality (LMI) format which can be conveniently solved by semidefinite programming techniques. Robust dissipativity results are subsequently derived for the case of convex polytopic uncertainties. Next the problem of state-feedback dissipativity is examined using state-feedback control for both delay-independent and delay-dependent cases. Numerical examples are presented to illustrate the theoretical developments.

Notations and facts: In the sequel, the Euclidean norm is used for vectors. We use W^t, W^{-1} to denote, respectively, the transpose, the inverse of any square matrix W and $W>0$ ($W<0$) stands for a symmetrical and positive- (negative-) definite matrix W. The n-dimensional Euclidean space is denoted by $\Re^{n\times n}$ and I stands for unit matrix with appropriate dimension. The symbol $\bullet$ will be used in some matrix expressions to induce a symmetric structure, that is if given matrices $L=L^t$ and $R=R^t$ of appropriate dimensions, then

$$\begin{bmatrix} L & \bullet \\ N & R \end{bmatrix} = \begin{bmatrix} L & N^t \\ N & R \end{bmatrix}.$$

Sometimes, the arguments of a function will be omitted when no confusion can arise.

2. A class of nonlinear time-delay system

We consider the following class of uncertain continuous-time-delay systems with parametric uncertainties and nonlinear perturbations:

$$\dot{x}(t) = A_0 x(t) + A_d x(t-\tau) + B_0 u(t) + \Gamma_0 w(t) + f(x(t),t) + g(x(t-\tau),t), \tag{2.1}$$

$$z(t) = G_0 x(t) + G_d x(t-\tau) + D_0 u(t) + \Phi_0 w(t), \tag{2.2}$$