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Exponential stability and stabilization of a class of uncertain linear time-delay systems

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Abstract

This paper presents new exponential stability and stabilization conditions for a class of uncertain linear time-delay systems. The unknown norm-bounded uncertainties and the delays are time-varying. Based on an improved Lyapunov–Krasovskii functional combined with Leibniz–Newton formula, the robust stability conditions are derived in terms of linear matrix inequalities (LMIs), which allows to compute simultaneously the two bounds that characterize the exponential stability rate of the solution. The result can be extended to uncertain systems with time-varying multiple delays. The effectiveness of the two stability bounds and the reduced conservatism of the conditions are shown by numerical examples.

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Keywords: Time-delay system; Uncertainty; Exponential stability; Stabilization; Linear matrix inequalities

1. Introduction

The stability analysis of linear time-delay systems is a topic of great practical importance, which has attracted a lot of interest over the decades, e.g. see [1,4,6,9,14,16,20,21]. Also, system uncertainties arise from many sources such as unavoidable approximation, data errors and aging of systems and so the stability issue

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of uncertain time-delay systems has been investigated by many researchers [3,13,15,19,22], where the Lyapunov–Krasovskii functional method is certainly used as the main tool. However, the conditions obtained in these papers only guarantee the asymptotic stability and must be solved upon a grid on the parameter space, which results in testing a finite number of linear matrix inequalities (LMIs). In this case, the results using finite griding points are unreliable and the numerical complexity of the tests grows rapidly. In [5,7,24], to reduce the conservatism of the stability condition the authors proposed a legitimate Lyapunov–Krasovskii functional which employs free weighting matrices. While most papers provided conditions for asymptotic stability and the uncertainties were not considered, it is interesting and important to find estimates of the exponential bounds for solutions of uncertain linear time-delay systems. An LMI-based approach combined the change of variable $\xi(t) = e^{\alpha t}x(t)$ is proposed in [8,12,15] for exponential stability estimates of systems with time-invariant delays. Using an appropriate Lyapunov–Krasovskii functional, the papers [11,18,23] proposed new exponential conditions in terms of standard LMI-types and has shown that the proposed method is less conservative than one in [10,12] through examples. More recently, by using general Halanay inequality and M-matrix theory, several new sufficient conditions are obtained in [17] to ensure the exponential robust stability of equilibrium point for time-delay systems.

In this paper, we consider the exponential stability problem of linear systems with time-varying delays and norm-bounded time-varying parameter uncertainties. To reduce the conservatism of the stability conditions, an improved Lyapunov–Krasovskii functional combined with Leibniz–Newton formula is introduced. The distributed delay free-weighting matrix functional combined with the Leibniz–Newton formula avoids the restriction on the derivative of time-varying delay and all the negative terms in the derivative of the Lyapunov–Krasovskii functional are retained. Moreover, the conditions obtained in this paper are also formulated in terms of LMIs [11], which can be efficiently solved by using various convex optimization algorithm [2]. Compared with the conditions obtained in [8,10–12,23], the introduction of extra variables and the use of the new improved Lyapunov–Krasovskii functional combined with Leibniz–Newton formula in our paper can reduce the conservatism in searching for the maximal decay rate such that the system is exponentially stable. The result can be extended to the uncertain system with time-varying multiple delays as well as to derive new sufficient conditions for robust exponential stabilization.

The paper is organized as follows. After Introduction, in Section 2 we give notations, definitions and technical lemmas needed for the proof of the main results. Sufficient conditions for the exponential stability and stabilization of the systems are presented in Section 3. Numerical examples to illustrate the effectiveness of our conditions in comparison with others are given in Section 4. The paper ends with conclusions and cited references.

2. Preliminaries

The following notations will be used throughout this paper: R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional space with the scalar product $\langle .,. \rangle$ and the vector norm $\|.\|$; $R^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimension. A^T denotes the transpose of A; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$;