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Nitsche-XFEM for the coupling of an incompressible fluid with immersed thin-walled structures

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Abstract

In this paper we introduce a Nitsche-XFEM method for fluid–structure interaction problems involving a thin-walled elastic structure (Lagrangian formalism) immersed in an incompressible viscous fluid (Eulerian formalism). The fluid domain is discretized with an unstructured mesh not fitted to the solid mid-surface mesh. Weak and strong discontinuities across the interface are allowed for the velocity and pressure, respectively. The fluid–solid coupling is enforced consistently using a variant of Nitsche's method with cut-elements. Robustness with respect to arbitrary interface intersections is guaranteed through suitable stabilization. Several coupling schemes with different degrees of fluid–solid time splitting (implicit, semi-implicit and explicit) are investigated. A series of numerical test in 2D, involving static and moving interfaces, illustrates the performance of the different methods proposed.

Keywords: Fluid–structure interaction; Immersed thin-walled structure; Unfitted meshes; XFEM; Nitsche method; Coupling schemes

1. Introduction

Mathematical models describing the mechanical interaction of an incompressible viscous fluid with an immersed thin-walled flexible structure appear in a wide variety of engineering fields: from micro-encapsulation to the aeroelasticity of parachutes and sailing boats (see, e.g., [1–3]). Such multi-physics systems are also particularly ubiquitous in nature. One can think, for instance, of the wings of a bird interacting with the air, the fins of a fish moving through the water, or the opening/closing dynamics of heart valves when blood is propelled into the arteries (see, e.g., [4–6]). The solid is deformed under the action of the fluid and the fluid flow is disturbed by the moving solid.

These problems are generally modeled by heterogeneous (parabolic/hyperbolic) systems of equations with different types of constitutive and geometrical non-linearities. This complicates the analysis both from the mathematical and numerical standpoint. In addition, the thin-walled nature of the immersed solid introduces jumps on the fluid stresses which, respectively, results in weak and strong discontinuities of the velocity and pressure fields. Standard finite

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element approximations, not allowing for such discontinuities, are known to deliver suboptimal convergence behavior and spurious numerical oscillations in the vicinity of the immersed solid (see, e.g., [7–9]).

The discontinuous features of the fluid solution can be straightforwardly incorporated within a standard finite element approximation by considering fitted fluid–solid meshes. It is well known, however, that maintaining fitted meshes may be cumbersome or unfeasible in presence of large interface deflections and topological changes (e.g., due to contacting solids). Though a number of advanced mesh update techniques have been reported in the literature (see, e.g., [10–14]), the favored alternative is to consider an unfitted mesh formulation, in which the fluid–structure interface moves independently of a background fluid mesh. Among these approaches, we can mention the Immersed Boundary/Fictitious Domain methods (e.g., [15–18,7,6,19,8,9]) and the methodologies based on a fully Eulerian description of the problem (e.g., [20,21]). In general, these methods are known to be inaccurate in space due to the continuous nature of the fluid approximations across the interface or to the discrete treatment of the interface conditions. The current trend to overcome these consistency issues is to combine a local XFEM enrichment with a cut-FEM methodology and a Lagrange multiplier treatment of the interface coupling (see, e.g., [22–25]). The price to pay, with respect to the original IB and FD methods, is the need of a specific tracking of the interface intersections (see, e.g., [26–28]) and a loss of robustness with respect to how the interface intersects the background fluid mesh (see, e.g., [29,30]).

A well-known alternative to the discrete treatment of the interface conditions via Lagrange multipliers is Nitsche's method (see, e.g., [31–33]). Because of its flexibility and mathematical soundness, the Nitsche mortaring has been applied to the design of numerical methods for a number of interface problems, including XFEM for elasticity [34–36], XFEM for two-phase transport problems [37,38], XFEM for incompressible flow [39] and robust and accurate FD methods for elliptic and mixed problems [40–42]. Nitsche's method was first applied to fluid–structure interaction problems with fitted meshes in [43] and used to design stable explicit coupling (or loosely coupled) schemes in [44,45]. It has recently been extended to fluid–structure interaction problems with unfitted meshes in [46], yielding robust and optimal a priori error estimates (fixed interface). In [46], the case of the coupling with thin-walled solids is limited to structures surrounding the fluid domain (i.e., not immersed).

The first contribution of this paper consists in the introduction of a robust and accurate Nitsche-XFEM method for fluid–structure interaction problems involving a thin-walled elastic structure immersed in an incompressible viscous fluid. We consider an Eulerian description for the fluid and a Lagrangian formulation for the solid. The fluid domain is discretized with an unstructured mesh not fitted to the solid mid-surface deformed mesh. In this unfitted mesh framework, the (strong) consistency of the proposed fluid–solid coupling builds on the following two ingredients:

- across the interface, locally enriched piecewise affine fluid velocity and pressure approximations respectively allow for weak and strong discontinuities (using the XFEM approach of [35,36]);
- the kinematic/dynamic fluid–solid coupling is enforced through a fluid-sided Nitsche's mortaring (based on [46]).

Besides, consistent symmetric stabilization operators are added to guarantee robustness with respect to arbitrary interface/element intersections (see, e.g., [42]) and to circumvent the classical inf–sup and convective related instabilities (see, e.g., [47–49,39]). In this regard, it is worth noting that for robustness these operators act on the fictitious region of the computational domain, without compromising the overall optimal accuracy of the method (in the energy norm).

The second contribution has to do with the time-discretization. Several coupling schemes with different levels of fluid–solid splitting are proposed: implicit, explicit and semi-implicit. The stability and convergence properties of the resulting fully discrete methods are analyzed within a representative linear setting (static interfaces). The salient features of the semi-implicit schemes introduced in this paper are twofold: (i) they deliver superior stability and accuracy with respect to alternative methods of explicit nature (see, e.g., [19]); (ii) they avoid the strong coupling of alternative semi-implicit coupling schemes (see, e.g., [50,9]) without compromising stability and accuracy.

Finally, the theoretical findings are substantiated by a series of numerical examples in 2D, involving static and moving interfaces, which illustrate the performance of the methods proposed by comparing with analytic solutions and fitted mesh approaches.

The rest of the paper is organized as follows. Section 2 is devoted to the derivation and the analysis of the methods within a linear setting (fixed interface). The space semi-discrete Nitsche-XFEM formulation is introduced in Section 2.1, Section 2.2 presents the time discretization and the different coupling schemes. In Section 3, the numerical methods are formulated within a non-linear setting involving moving interfaces. Numerical evidence illustrating the